

# ON THE ROLE OF VENTRICLES IN DIFFUSE AXONAL INJURIES

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## INTRODUCTION

For over fifty years the theory of viscoelasticity has played a major role in modeling brain injuries. The main premise of this approach is that the brain is injured when the strain field, created in the brain tissue by shear waves, assumes sufficiently high values. In particular, the linear Voigt PDE system describing the motion of shear waves in viscoelastic solids has been successfully used in modeling closed head injuries such as hematomas [1] or diffuse axonal injuries (DAI) [2].

Whereas it is generally accepted that a 50% stretching of neurons can cause hematomas [3], the exact mechanism of DAI is unclear. The prevailing theory is that a 20%-30% stretching of a neuron's membrane leads to a chemical 'poisoning' that causes neuronal death around twelve hours after the accident [4]. Mechanically, neurons might be able to 'survive' even 80% stretching [5].

However, the basic deficiency of the linear Voigt model, as well as other models known to us, is that they cannot explain the following three key features of DAI [4,6]:

1. The highly localized character of the damage – some neurons are affected while close neighbors are not;
2. These 'point-wise' injuries of neurons are scattered over large regions, principally in the white matter;
3. The injuries are concentrated at the white-matter/ventricle and gray-matter/white-matter boundary.

From a mathematical point of view, it is rather impossible to model such features without considering the fluid (nonlinear) aspect of the brain tissue, which consists of 80% of water. One recent, nonlinear model attempts to do so by treating the brain tissue as a viscoelastic, porous, solid structure permeated with water-like fluid [7].

## NONLINEAR, FLUID MODEL OF CLOSED HEAD INJURIES

In our approach to modeling CHI, we have used the viscoelastic Voigt model as a starting point and incorporated the fluid aspect of the brain tissue by replacing the linear partial temporal derivative in the Voigt PDE system with the corresponding nonlinear material derivative. Specifically, we have introduced the following system of PDEs to describe the motion within the brain in traumatic situations:

$$\frac{D\mathbf{v}}{Dt} = -\nabla\phi + c^2\Delta\mathbf{u} + \nu\Delta\mathbf{v}, \quad \frac{D\mathbf{u}}{Dt} = \mathbf{v}, \quad \nabla \cdot \mathbf{v} = 0. \quad (1)$$

Here  $D/Dt \equiv \partial/\partial t + \mathbf{v} \cdot \nabla$  is the material derivative with  $\mathbf{v}(\mathbf{x}, t)$  being the velocity vector,  $\mathbf{u}(\mathbf{x}, t)$  is the Lagrangian displacement vector of a material parcel labeled by its initial position  $\mathbf{x}_0$ ,  $\phi$  denotes a scalar potential composed of density-normalized pressure and hydrostatic compression,  $c$  is the velocity of shear waves, and  $\nu$  is the kinematic viscosity coefficient. In other words, we have augmented the standard Navier-Stokes PDEs for incompressible fluids with an additional forcing proportional to the Lagrangian integrals of the flow velocity. This amounts to treating the brain tissue as a non-Newtonian fluid.

Our postulate of introducing the nonlinear material derivative to link the velocity  $\mathbf{v}$  and the displacement  $\mathbf{u}$  enables the fluidity of brain tissue while retaining its elastic-solid tendency to return to its initial form once the deforming force is eliminated. Indeed, integrating the velocity of a material parcel along its trajectory to find its displacement allows the parcel to 'remember' where it came from.

To account for free boundaries between various brain structures, in particular between the white matter and the ventricles, we treat  $c$  and  $\nu$  as material constants, i.e., we assume they satisfy:

$$Dc/Dt = 0, \quad D\nu/Dt = 0. \quad (2)$$

The finite-difference method we have used to solve (1) and (2) is rooted in computational studies of stratified, rotating fluids in the research area of atmospheric dynamics. More precisely, we adapted the fluid code EULAG developed by Dr. P. Smolarkiewicz at the National Center for Atmospheric Research in Boulder, CO [8].

Our previous numerical simulations (see [9-11] and references herein) showed that when a rapid head rotation leads to a material velocity of the brain tissue that is higher than the velocity of shear waves in the tissue, large and highly localized maxima of strain can appear due to nonlinear phenomena such as the steepening of wave fronts. We also showed that since the gray matter shear modulus is greater than the white matter shear modulus [12,13], 'harmless' shear waves in the gray matter can still induce these nonlinear phenomena in the white matter, leading to DAI at the gray/white-matter boundary.

## DIFFUSE AXONAL INJURIES NEAR VENTRICLES

Since the cerebral fluid does not support shear waves, ventricles with their free boundaries should mitigate brain injuries rather than cause them by playing the role of energy absorbers. In fact, a significant strain reduction in the white matter near the ventricle has recently been observed due explicitly to the free boundary motion [14]. Nevertheless, we hypothesized that ventricles can also transfer or reflect energy of shear waves in a similar way as is done by the liquid core of the Earth during earthquakes. Consequently, a highly localized accumulation of energy could appear, leading to DAI.

To verify our hypothesis, we tested dynamic responses of brain material during simulated rotations of idealized 2-D cross-sections of the brain. In particular, we mimicked a rotation of an ellipse (with the half-axes equal to 0.1m and 0.075m) containing a 10% layer of the gray matter and a 90% layer of the white matter. A small sub-region (representing an idealized cross-section of a lateral ventricle) was filled with cerebral fluid. To model the flow in the ventricle, we used the standard Navier-Stokes equations, i.e., we set  $c=0$  in (1).

Fig. 1 depicts the case where the ellipse is impulsively rotated for 0.05s with a tangential velocity of 0.8m/s so that the material velocity induced in the brain tissue is smaller than the 0.9m/s velocity of the 'white' shear waves and the 1.8m/s velocity of the 'gray' waves [12]. The graph shows the values of the operator norm  $N$  of the symmetric part of the displacement's Jacoby matrix  $du/dx$ , evaluated relative to the rotation of the ellipse at  $t=0.1s$ , i.e., the 'effective' values of the strain. The high values of  $N$  in the ventricle indicate that the energy of 'harmless' shear waves induced in the white matter has been transferred through the boundary, leading to a turbulent flow there.

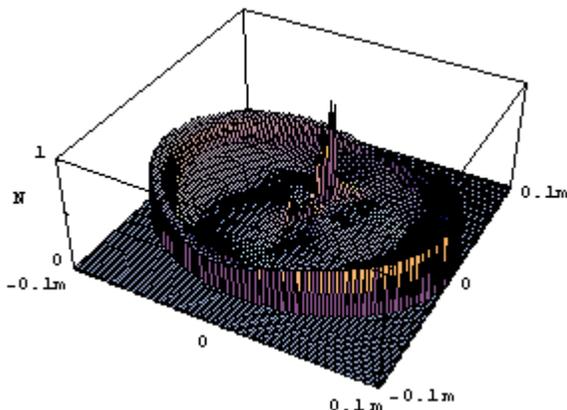


Fig. 1. Strain norm in the brain tissue and the ventricle.

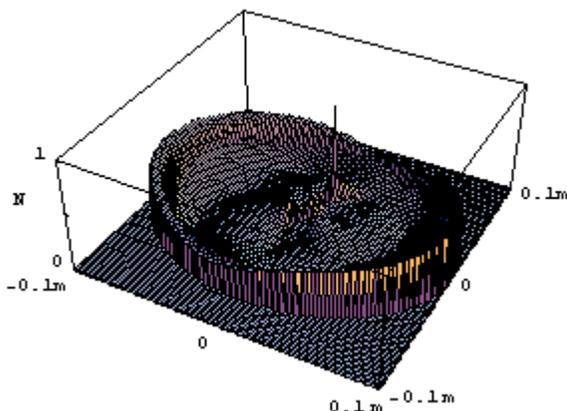


Fig. 2. Strain norm outside the ventricle.

Fig. 2 presents results of the same simulation with the values of  $N$  set to zero inside the ventricle. This enables us to show that the turbulent flow in the ventricle induces 'harmful' shear waves in the neighboring white matter, and that these waves continue to spread after the rotation stops. Indeed, 0.05s after the rotation stops, the highly localized maximum of  $N$  in the white matter is sufficiently large to stretch neurons by 90% and, consequently, to cause DAI.

Assuming that boundaries are rigid, i.e., eliminating (2) does not change the basic features of this specific solution but leads to larger values of  $N$  – especially in the ventricle. The results indicate that our nonlinear fluid viscoelastic model is capable of replicating the three key features of DAI listed above and provides a reasonable explanation of why DAI occur predominantly in the white matter at the white-matter/ventricle and white-matter/gray-matter boundaries.

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