Lower Bounds for Sorting

- Sorting by comparison of keys
  - input consists of n distinct keys, \( a_1, a_2, \ldots, a_n \)
  - all comparisons have the form \( a_i \leq a_j \)
  - comparisons are the only way to get order information

Decision-Tree Model

- internal node has the form \( a_i : a_j \)
  - \( a_i \leq a_j \) is tested
  - branch left if \( a_i \leq a_j \)
  - branch right if \( a_i > a_j \)
- leaf contains a permutation that sorts the input
- each execution corresponds to a path from root to leaf

- Model
  - abstraction of sorting
  - represents comparisons
  - all other information is ignored

- Theorem: Any decision tree that sorts \( n \) elements has height \( \Omega(n \log n) \).