Review

Overview

- **Theory of computation**: central areas:

  Automata, Computability, Complexity

- **Computability**: Is the problem solvable?
  - solvable and unsolvable

  ↓
- **Complexity**: Is the problem easy to solve?
  - easy ones and hard ones

- Both deal with formal models of computation: **Turing machines**, recursive functions, lambda calculus, and production systems

Overview

- **Automata theory**: study of abstract machines and problems they are able to solve.
  - closely related to formal language theory as the automata are often classified by the class of formal languages they are able to recognize.
  - An abstract machine, also called an abstract computer, is a theoretical model of a computer hardware or software system
  - FSM, PDA, Turing machine

- **Formal language**: A set of strings over a given alphabet.
  - In contrast to natural language
  - Often defined by formal grammar
  - Regular, context-free, D, SD

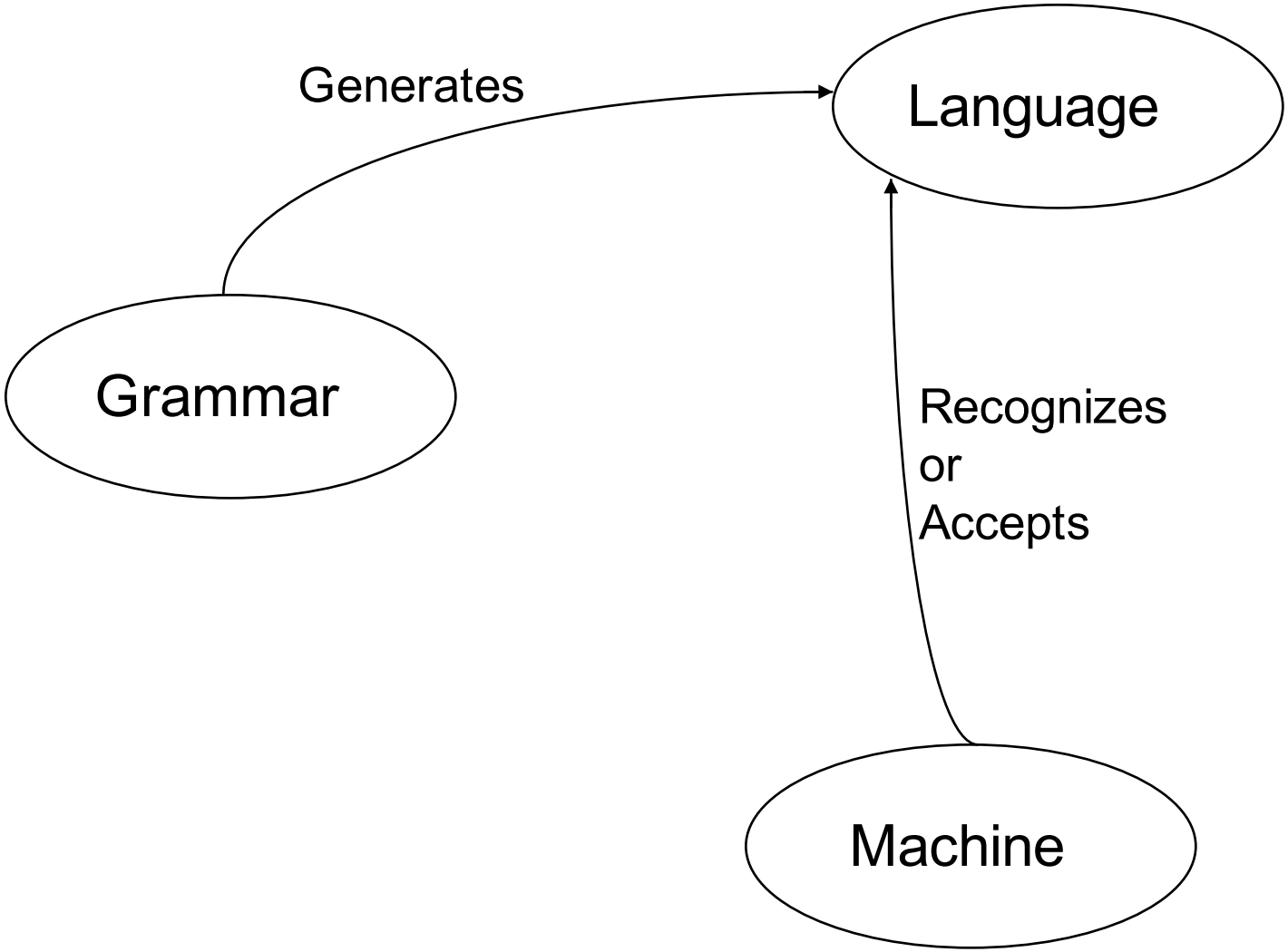
Approach

- Language recognition framework:
  - transform a problem into a decision problem (verification) if it is not
  - encode the inputs as strings and then define a language that contains exactly the set of inputs for which the desired answer is yes
  - the original problem now becomes a language recognition problem

- Now, all problems have the same look

- Machines (FSM, PDA, TM …) are abstract computational models. They are used to classify problems.

Grammars, Languages, and Machines



Path

- Given input w, a maximal sequence of moves that M takes from the starting configuration
- A path P ends when it enters an accepting configuration, or there is no transition defined for its current configuration, i.e., it has no where to go.
- If P ends in an accepting configuration, P accepts w.
- If P ends in a non-accepting configuration, P rejects w.
- If there is some path of M that accepts w, M halts and accepts w.
- If all paths of M reject w, M halts and rejects w.
- It is possible that M neither accepts nor rejects w. In which case, none of its paths accepts w and some path does not reject w.

- Applies to all kinds of automata in the scope of our study.

Different from the text

In Particular

- If M is deterministic, there's only **one** path.

- For DFSM and DPDA, the path is finite and guaranteed to end. Upon halting, it either accepts or rejects. So, DFSM and DPDA are guaranteed to halt (in at most |w| moves).
- Since DFSM = regular languages, the above halting property of DFSM implies that regular languages are in D.
- For DTM, the path maybe infinite. So DTM may not halt for some w.
    - It happens when M is a semi-decider and not a decider, and for w that is not in L(M)
    - Not that DTM = SD

In Particular

- If M is nondeterministic, there're maybe multiple paths.
- For NDFSM and NDPDA, if without ε-transitions, they are guaranteed to halt in at most |w| moves. Otherwise, no.
- For each NDFSM M, there is an equivalent DFSM M'
    - "equivalent" means L(M) = L(M')
- For each NDPDA M, there an equivalent NDPDA M' that halts.
- Since NDPDA = context-free languages, the above halting property of NDPDA implies that context-free languages are in D.
- For NDTM, no such properties, as DTM does not even always halt.

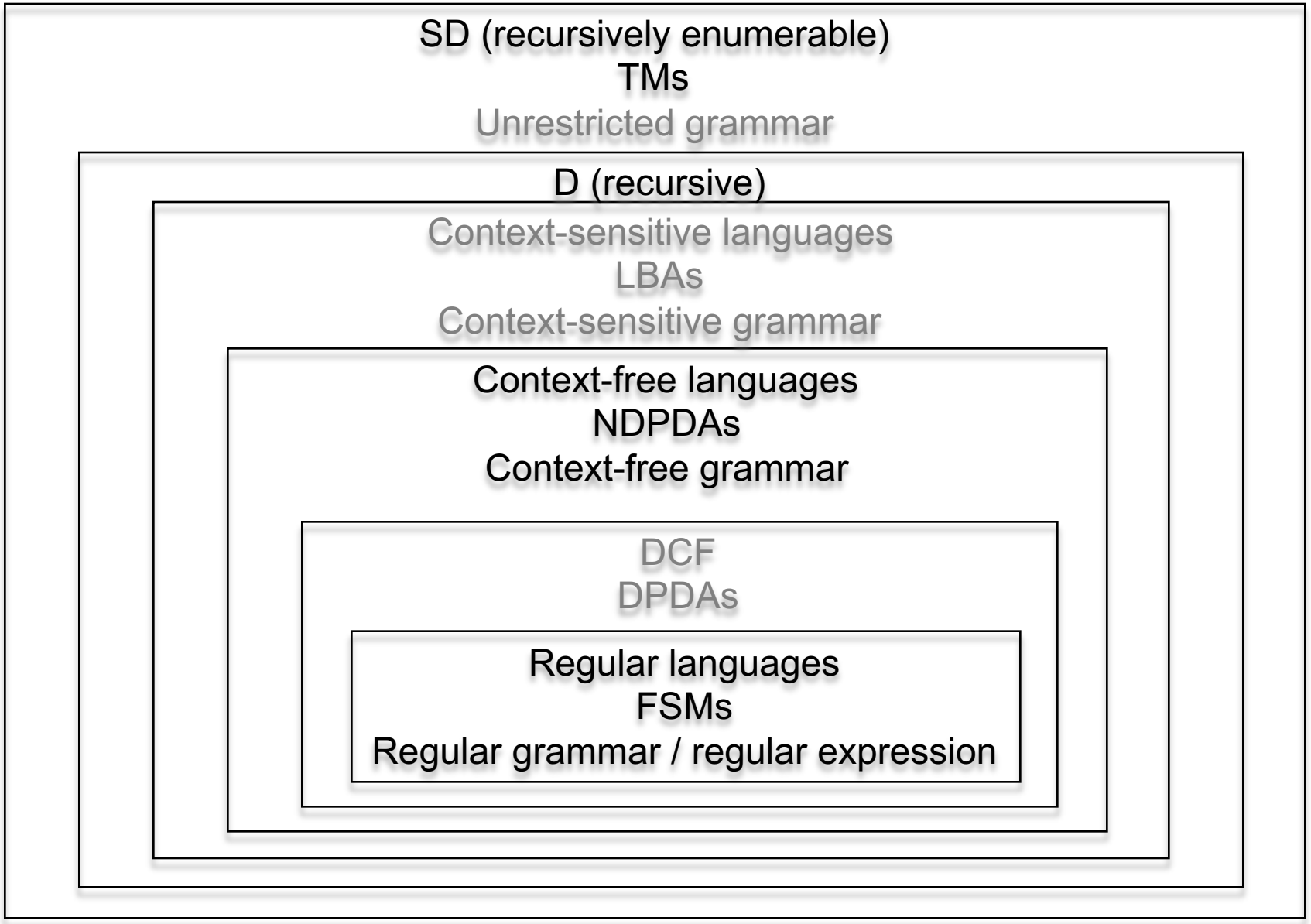
Recognizer and Decider

- L(M): language accepted by M.
  - For each $w \in L(M)$, M halts

• Recognizer: M is a recognizer (acceptor) of L if M halts on any $w \in L$ and returns "yes"
  - They are semi-deciders

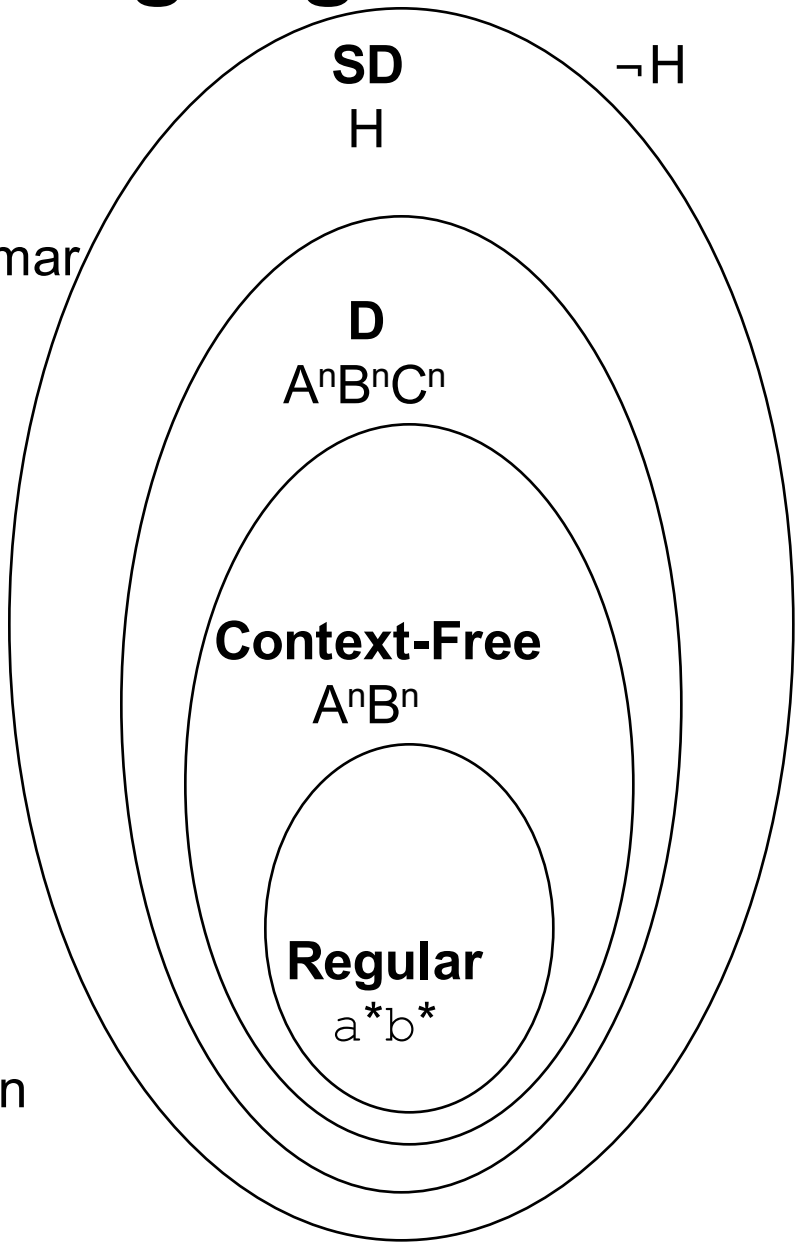
• Decider: M is a decider of L if M always halts, returns "yes" if $w \in L$ and "no" if $w \notin L$

Languages, Machines, and Grammars



Language Summary

IN		OUT
Semideciding TM	SD: H	Reduction
Enumerable		
Unrestricted grammar		
Deciding TM	D: $A^nB^nC^n$	Diagonalization
Lexic. enum		Reduction
L and ¬*L* in SD		
CF grammar	Context-Free: A^nB^n	Pumping
PDA		Closure
Closure		
Regular grammar	Regular: a*b*	Pumping
Regular Expression		Closure
FSM		
Closure		



Reduction

$P_{old} \leq P_{new}$

- means that P_{old} is reducible to P_{new}
- reduction from L_{old} to L_{new}
- if L_{new} can be done, L_{old} can be done

1. Known P_{old} is not in D, we can show P_{new} is not in D
2. Known P_{old} is not in SD, we can show P_{new} is not in SD
- For both, no need to care about the efficiency of the reduction.

Can also be used in complexity to show NP-hardness:

3. Known P_{old} is NP-hard, we can show P_{new} is NP-hard
- Need to care about the efficiency of the reduction (polynomial).

Mapping reduction: the most straightforward way of reduction is to transform instances of L_{old} into instances of L_{new}

L_{old} is *mapping reducible* to L_{new} ($L_{old} \leq_M L_{new}$) iff there exists some computable function f such that:

$$\forall x \in \Sigma^* \; (x \in L_{old} \leftrightarrow f(x) \in L_{new})$$

The Complexity Zoo

The attempt to characterize the decidable languages by their complexity:

http://qwiki.stanford.edu/wiki/Complexity_Zoo

Notable ones:

P: solution found by deterministic TM (algorithm) in polynomial time

- tractable
- context-free (including regular) languages are in P

NP: solution found by nondeterministic TM (algorithm) in polynomial time

- solution can be verified by DTM in polynomial time

NP-complete: as hard as any one in NP & in NP (hardest ones in NP)

- no efficient algorithm is known
- require non-trivial search, as in TSP

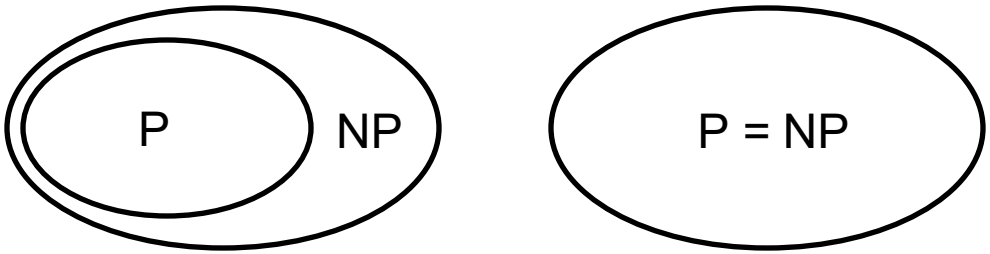
NP-hard: as hard as any one in NP (not necessarily in NP)

- L is NP-complete if it is in NP and it is NP-hard
- Intractable = not in P. But since it's believed P ≠ NP, loosely intractable = NP hard = NP complete + not in NP
- If it's proved that P = NP, then intractable = not in P = not in NP

P and NP

Greatest unsolved problem in theoretical computer science:
Is P = NP? ***The Millenium Prize***

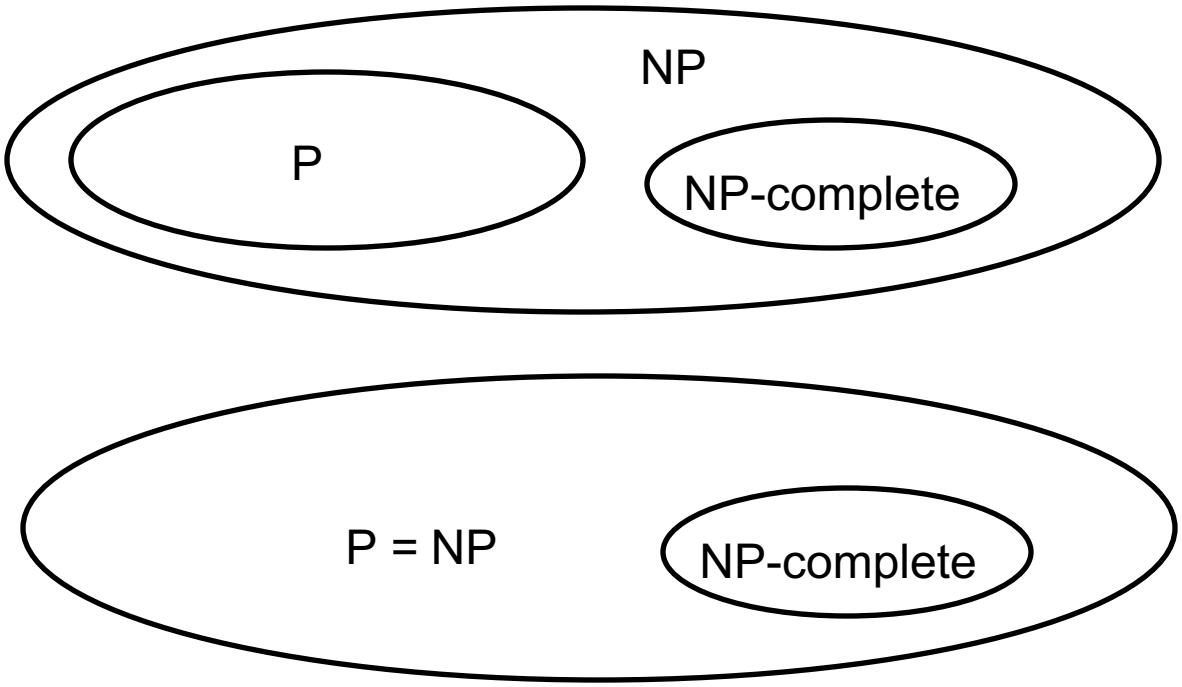
Two possibilities:



If P = NP, any polynomially verifiable problems would be polynomially decidable

NP-Completeness

- The class of NP-complete is important, many of its members, like TSP, have substantial practical significance.
- Two possibilities:



Strategy for Proving NP-completeness of L_{new}

- Show that L_{new} belongs to NP
  - Exhibit an NDTM to decide it in polynomial time

    Or, equivalently,
  - Exhibit a DTM to verify it in polynomial time

  - This establishes an upper bound on the complexity of L_{new}

- Show that L_{new} is NP-hard by finding another NP-hard language L_{old} such that

$$L_{old} \leq_P L_{new}$$

  - This establishes a lower bound on the complexity of L_{new}

Example Reductions