Hash Tables

CS 5301
Fall 2013
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Main + Savitch: chapter 12.2-3
Weiss: chapter 20

What are hash tables?

- A Hash Table is used to implement a set (or a search table), providing basic operations in constant time:
  - insert
  - remove (optional)
  - find
  - makeEmpty (need not be constant time)
- It uses a function that maps an object in the set (a key) to its location in the table.
- The function is called a hash function.

Using a hash function

HandyParts company makes no more than 100 different parts. But the parts all have four digit numbers.

This hash function can be used to store and retrieve parts in an array.

Hash(partNum) = partNum % 100

Placing elements in the array

Use the hash function

Hash(partNum) = partNum % 100

to place the element with part number 5502 in the array.
### Placing elements in the array

<table>
<thead>
<tr>
<th>Index</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Empty</td>
</tr>
<tr>
<td>1</td>
<td>4501</td>
</tr>
<tr>
<td>2</td>
<td>5502</td>
</tr>
<tr>
<td>3</td>
<td>7803</td>
</tr>
<tr>
<td>4</td>
<td>Empty</td>
</tr>
<tr>
<td>97</td>
<td></td>
</tr>
<tr>
<td>98</td>
<td>2298</td>
</tr>
<tr>
<td>99</td>
<td>3699</td>
</tr>
</tbody>
</table>

Next place part number 6702 in the array.

\[
\text{Hash(partNum) = partNum} \mod 100
\]

\[
6702 \mod 100 = 2
\]

But values[2] is already occupied.

**COLLISION OCCURS**

### How to resolve the collision?

One way is by linear probing. This uses the following function

\[
(\text{HashValue} + 1) \mod 100
\]

repeatedly until an empty location is found for part number 6702.

### Resolving the collision

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<tr>
<td>4</td>
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<td>98</td>
<td>2298</td>
</tr>
<tr>
<td>99</td>
<td>3699</td>
</tr>
</tbody>
</table>

Still looking for a place for 6702 using the function

\[
(\text{HashValue} + 1) \mod 100
\]

### Collision resolved

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<thead>
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<td>2298</td>
</tr>
<tr>
<td>99</td>
<td>3699</td>
</tr>
</tbody>
</table>

Part 6702 can be placed at the location with index 4.
Collision resolved

Part 6702 is placed at the location with index 4.

Where would the part with number 4598 be placed using linear probing?

Hashing concepts

- **Hash Table**: where objects are stored by according to their key (usually an array)
  - key: attribute of an object used for searching/sorting
  - number of valid keys usually greater than number of slots in the table
  - number of keys in use usually much smaller than table size.

- **Hash function**: maps keys to a Table index

- **Collision**: when two separate keys hash to the same location

Hashing concepts

- **Collision resolution**: method for finding an open spot in the table for a key that has collided with another key already in the table.

- **Load Factor**: the fraction of the hash table that is full
  - may be given as a percentage: 50%
  - may be given as a fraction in the range from 0 to 1, as in: .5

Hash Function

- **Goals**:
  - computation should be fast
  - should minimize collisions (good distribution)

- **Some issues**:
  - should depend on ALL of the key (not just the last 2 digits or first 3 characters, which may not themselves be well distributed)
Hash Function

- Final step of hash function is usually: temp % size
  - temp is some intermediate result
  - size is the hash table size
  - ensures the value is a valid location in the table
- Picking a value for size:
  - Bad choices:
    - a power of 2: then the result is only the lowest order bits of temp (not based on whole key)
    - a power of 10: result is only lowest order digits of decimal number
  - Good choices: prime numbers

Collision Resolution: Linear Probing

- Insert: When there is a collision, search sequentially for the next available slot
- Find: if the key is not at the hashed location, keep searching sequentially for it.
  - if it reaches an empty slot, the key is not found
- Problem: if the the table is somewhat full, it may take a long time to find the open slot.
- Problem: Removing an element in the middle of a chain

Linear Probing: Example

- Insert: 89, 18, 49, 58, 69, \( \text{hash(k)} = k \mod 10 \)

<table>
<thead>
<tr>
<th>Empty Table</th>
<th>After 89</th>
<th>After 18</th>
<th>After 49</th>
<th>After 58</th>
<th>After 69</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td>49</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>69</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>4</td>
<td></td>
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<tr>
<td>5</td>
<td></td>
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<tr>
<td>6</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>18</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>89</td>
<td>89</td>
<td>89</td>
<td>89</td>
<td>89</td>
</tr>
</tbody>
</table>

Probing function (attempt i): \( h(K) = (\text{hash}(K) + i) \mod \text{tablesize} \)

- 49 is in 0 because 9 was full
- 58 is in 1 because 8, 9, 0 were full
- 69 is in 1 because 9, 0 were full

Collision Resolution: Separate chaining

- Use an array of linked lists for the hash table
- Each linked list contains all objects that hashed to that location
  - no collisions
Separate Chaining

- To insert a an object:
  - compute hash(k)
  - insert at front of list at that location (if empty, make first node)

- To find an object:
  - compute hash(k)
  - search the linked list there for the key of the object

- To delete an object:
  - compute hash(k)
  - search the linked list there for the key of the object
  - if found, remove it