Sets & Hash Tables

Week 13

Weiss: 20
Main & Savitch: 3, 12.2-3

CS 5301
Fall 2014

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What are sets?

- A set is a collection of objects of the same type that has the following two properties:
  - there are no duplicates in the collection
  - the order of the objects in the collection is irrelevant.

- \{6,9,11,-5\} and \{11,9,6,-5\} are equivalent.

- There is no first element, and no successor of 9.

Set Operations

- Set construction
  - the empty set (0 elements in the set)

- isEmpty()
  - True, if the set is empty; false, otherwise.

- Insert(element)
  - If element is already in the set, do nothing; otherwise add it to the set

- Delete(element)
  - If element is not a member of the set, do nothing; otherwise remove it from the set.

Set Operations

- Member(element): boolean
  - True, if element is a member of the set; false, otherwise

- Union(Set1,Set2): Set
  - returns a Set containing all elements of the two Sets, no duplications.

- Intersection(Set1,Set2): Set
  - returns a Set containing all elements common to both sets.
Set Operations

- **Difference(Set1,Set2):** Set
  - returns a Set containing all elements of the first set except for the elements that are in common with the second set.
- **Subset(Set1,Set2):** boolean
  - True, if Set2 is a subset of Set1 (if all elements of the Set2 are also elements of Set1).

Implementation

- **Array of elements implementation**
  - each element of the set will occupy an element of the array.
  - the member (find) operation will be inefficient, must use linear search.
- **see Lab 6, exercise 1**
  - represented a set of integers, but called it a “Batch”
  - class contained a pointer to a dynamically allocated array of ints
- **Exercise:** implement all of the set operations for this set

Implementation

- **Boolean array implementation**
  - size of the array must be equal to number of all possible elements (the universe).
- **Here is the set {Monday, Wednesday, Friday}:**
  - if `daysOfWeek[1]` is true, then Monday is in the Set.

```cpp
int map(string x) {
    if (x=="Sunday") return 0;
    if (x=="Monday") return 1;
    if (x=="Tuesday") return 2;
    if (x=="Wednesday") return 3;
    if (x=="Thursday") return 4;
    if (x=="Friday") return 5;
    if (x=="Saturday") return 6;
}
```
- if `daysOfWeek[map("Monday")].` is true, then Monday is in the Set.
**Implementation**

- **Boolean array implementation: member**
  ```java
  bool member(string x) {
    int pos = map(x);
    if (0<=pos && pos<7)
      return daysOfWeek[pos];
    return false;
  }
  ```

- **Boolean array implementation: union**
  ```java
  // c will be the union of a and b:
  void union(bool a[], bool b[], bool c[]) {
    for (int pos=0; pos<7; pos++)
      c[pos] = (a[pos] || b[pos]);
  }
  ```

- Exercise: implement all of the set operations for the set implemented as a boolean array

**What are hash tables?**

- A Hash Table is used to implement a **set** (or a **search table**), providing basic operations in constant time:
  - insert
  - delete (optional)
  - find (also called “member”)
  - makeEmpty (need not be constant time)
- It uses a function that maps an object in the set (a key) to its location in the table.
- The function is called a **hash function**.

**Using a hash function**

**HandyParts company makes no more than 100 different parts. But the parts all have four digit numbers.**

This hash function can be used to store and retrieve parts in an array.

Hash(partNum) = partNum % 100

**Placing elements in the array**

Use the hash function

Hash(partNum) = partNum % 100
to place the element with part number 5502 in the array.
Placing elements in the array

Next place part number 6702 in the array.

Hash(partNum) = partNum % 100

6702 % 100 = 2

But values[2] is already occupied.

COLLISION OCCURS

Resolving the collision

Still looking for a place for 6702 using the function

(HashValue + 1) % 100

Collision resolved

Part 6702 can be placed at the location with index 4.
Collision resolved

Part 6702 is placed at the location with index 4.

Where would the part with number 4598 be placed using linear probing?

Hashing concepts

- **Hash Table**: (usually an array) where objects are stored according to their key
  - **key**: attribute of an object used for searching/sorting
  - number of **valid** keys usually greater than number of slots in the table
  - number of keys in use usually much smaller than table size.

- **Hash function**: maps a key to a Table index

- **Collision**: when two separate keys hash to the same location

Hashing concepts

**Collision resolution**: method for finding an open spot in the table for a key that has collided with another key already in the table.

**Load Factor**: the fraction of the hash table that is full
- may be given as a percentage: 50%
- may be given as a fraction in the range from 0 to 1, as in: .5

Implementation

- **Standard array implementation**
  - keys are ints:

```cpp
class HashTable {
private:
    struct Entry {
        int value;
        int status;  // 0 = open 1 = occupied 2 = deleted
    };
    Entry *array;  //array of elements
    int size;  //size of array
public:
    HashTable (int);  //initialize elements status to 0
    ~HashTable();
    bool find(int);  //return true if int in table
    void insert (int);  //add int to table
    void display();  //show elements in table
    void remove(int);  //remove int from table
};
```
Hash Function

- **Goals:**
  - Computation should be fast
  - Should minimize collisions (good distribution)

- **Some issues:**
  - Should depend on ALL of the key (not just the last 2 digits or first 3 characters, which may not themselves be well distributed)

- **Final step of hash function is usually:**
  - Temp is some intermediate result
  - Size is the hash table size
  - Ensures the value is a valid location in the table

Collision Resolution: Linear Probing

- **Insert:** When there is a collision, search sequentially for the next open slot
- **Find:** If the key is not at the hashed location, keep searching sequentially for it.
  - If it reaches an open slot, the key is not found
- **Remove:** If the key is not at the hashed location, keep searching sequentially for it.
  - If the key is found, set the status to open
- **Problem:** Removing an element in the middle of a chain. The Find method needs to know to keep searching to the end of the chain.

Remove Solution

- **Remove:** If the key is not at the hashed location, keep searching sequentially for it.
  - Skip deleted items and occupied items not matching the key
  - If the key is found, mark it as deleted.

- **Find:** If the key is not at the hashed location, keep searching sequentially for it.
  - Skip deleted items and occupied items not matching the key

- **Insert:** Use **Find**, if the key is NOT in the list: Start the search again:
  - Skip occupied items not matching the key
  - This way we can reuse the deleted spots

Linear Probing: Example

- **Insert:** 89, 18, 49, 58, 69, \( hash(k) = k \mod 10 \)

<table>
<thead>
<tr>
<th>Empty Table</th>
<th>After 89</th>
<th>After 18</th>
<th>After 49</th>
<th>After 58</th>
<th>After 69</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>49</td>
<td>49</td>
<td>49</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>58</td>
<td>58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>69</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
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<tr>
<td>4</td>
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<td>5</td>
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<tr>
<td>6</td>
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<td>7</td>
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</tr>
<tr>
<td>8</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>89</td>
<td>89</td>
<td>89</td>
<td>89</td>
<td>89</td>
</tr>
</tbody>
</table>

- 49 is in 0 because 9 was full
- 58 is in 1 because 8, 9, 0 were full
- 69 is in 1 because 9, 0 were full
Collision Resolution: Separate Chaining

- Use an array of linked lists for the hash table
- Each linked list contains all objects that hashed to that location
  - no collisions

Hash function is still:
\[ h(K) = k \mod 10 \]

Separate Chaining

- To insert an object:
  - compute hash(k)
  - insert at front of list at that location (if empty, make first node)
  - (do not insert if already in the list)
- To find an object:
  - compute hash(k)
  - search the linked list there for the key of the object
- To delete an object:
  - compute hash(k)
  - search the linked list there for the key of the object
  - if found, remove it

Implementation

- Array of linked lists implementation
  - The data structure:

```cpp
class HashTable {
    private:
        static const int SIZE = 100;
        struct Node {
            int key;
            node *nextNode;
        }
        Node* pTable[SIZE];  //array of pointers to Nodes
    }
    // constr should init all pointers in the array to NULL
```