Heaps
Chapter 21

CS 3358
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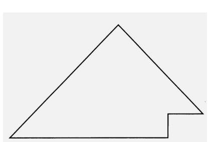
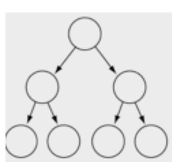
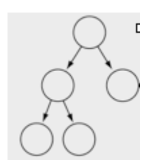
Binary heap data structure

- A binary heap is a special kind of binary tree
  - has a restricted structure (must be complete)
  - has an ordering property (parent value is smaller than child values)
  - NOT a Binary Search Tree!
- Used in the following applications
  - Priority queue implementation: supports enqueue and deleteMin operations in O(log N)
  - Heap sort: another O(N log N) sorting algorithm (see last slide).

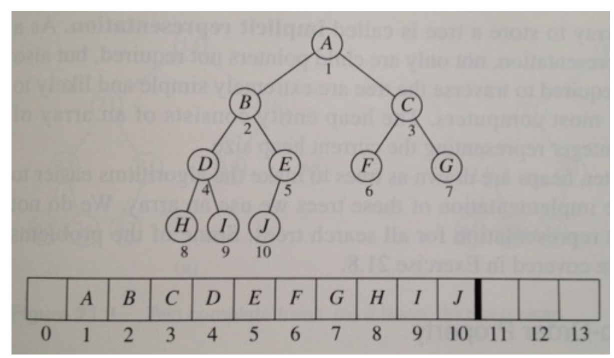
Binary Heap:
structure property

- **Complete binary tree**: a tree that is completely filled
  - every level except the last is completely filled.
  - the bottom level is filled left to right (the leaves are as far left as possible).

Complete Binary Trees

- A complete binary tree can be easily stored in an array
  - place the root in position 1 (for convenience)



Complete Binary Trees
Properties

- The height of a complete binary tree is floor($\log_2$ N)
  (floor = biggest int less than)
- In the array representation:
  - put root at location 1
  - use an int variable (size) to store number of nodes
  - for a node at position i:
    - left child at position `2i` (if 2i <= size, else i is leaf)
    - right child at position `2i+1` (if 2i+1 <= size, else i is leaf)
    - parent is in position `floor(i/2)` (or use integer division)

Binary Heap:
ordering property

- In a heap, if X is a parent of Y, value(X) is less than or equal to value(Y).
  - the minimum value of the heap is always at the root.
  - findMin() is O(1)

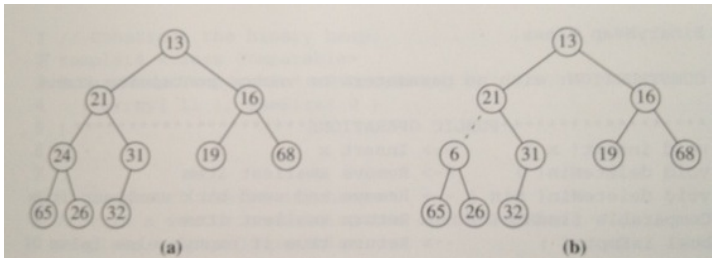


Figure 21.3 Two complete trees: (a) a heap; (b) not a heap.

Heap: insert(x)

- First: add a node to tree.
  - Maintain a complete tree: place item at next available location, size+1
- Next: maintain the ordering property:
  - if x is greater than its parent: done
  - else swap with parent, repeat
- Called “percolate up” or “reheap up”
- preserves ordering property
- O(log N) worst case

Heap: insert(x)

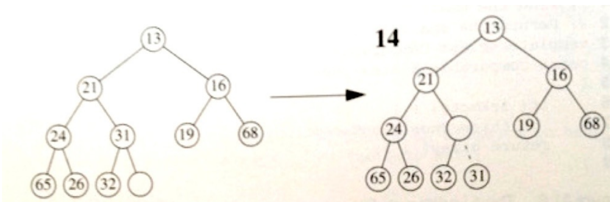


Figure 21.7 Attempt to insert 14, creating the hole and bubbling the hole up.

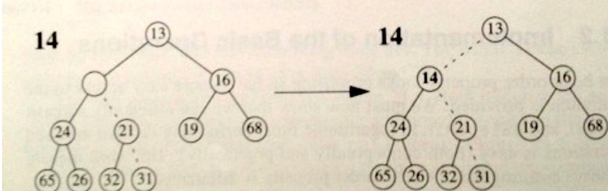


Figure 21.8 The remaining two steps required to insert 14 in the original heap shown in Figure 21.7.

Heap: deleteMin()

- Minimum is at the root, removing it leaves a hole.
- First: maintain complete tree: move last element up to the root.
- Next: maintain the ordering property, start with root:
  - if both children are greater than the parent: done
  - otherwise, swap the <u>smaller</u> of the two children with the parent, repeat (why not the larger one?)
- Called “percolate down” or “reheap down”
- preserves ordering property
- O(log N) worst case

Heap: deleteMin()

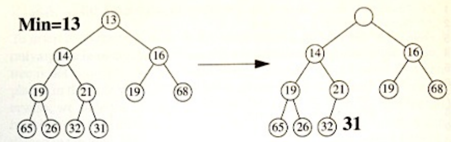


Figure 21.10 Creation of the hole at the root.

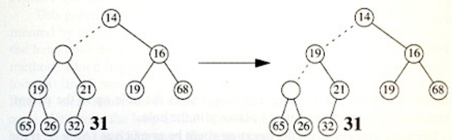


Figure 21.11 The next two steps in the `deleteMin` operation.

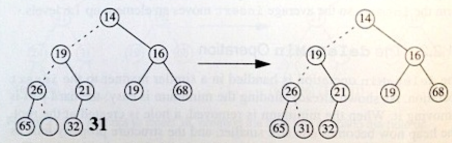


Figure 21.12 The Last two steps in the `deleteMin` operation.

Heap: buildHeap()

- buildHeap takes a tree that does not have heap order and establishes it.
- The algorithm works bottom-up:
  - when processing a given node, its two children will already be in heap order.
  - then we can use percolate down to put the current node in the right place, and preserve the heap order property.
- No need to apply to leaves.
- Turns out this algorithm is O(N) (see book for proof)
- N inserts using insert(x) would be O(N log N) worst case.

Heap: buildHeap()

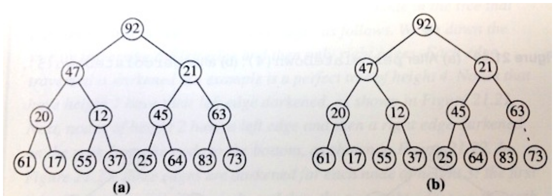


Figure 21.17 (a) Initial heap; (b) after `percolateDown(7)`.

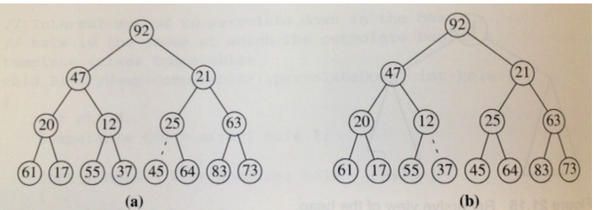


Figure 21.18 (a) After `percolateDown(6)`; (b) after `percolateDown(5)`.

Heap: buildHeap()

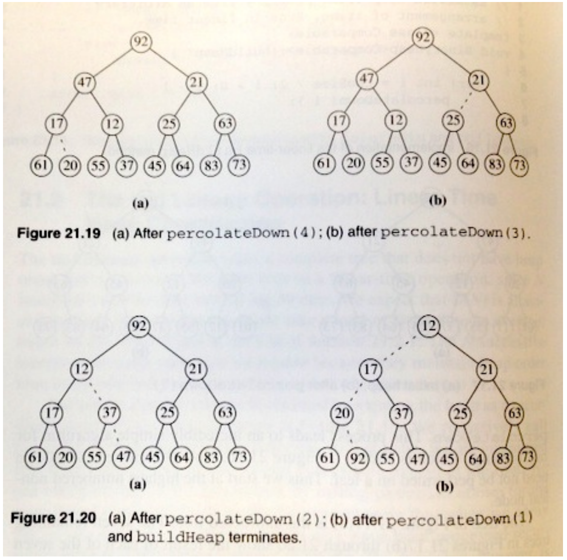


Figure 21.19 (a) After percolateDown(4); (b) after percolateDown(3).

Figure 21.20 (a) After percolateDown(2); (b) after percolateDown(1) and buildHeap terminates.

Heapsort

- Using a heap to sort a list:
  1. `insert` every item into a binary heap
  2. extract every item by calling `deleteMin` N times.
- Can make it slightly more efficient by using `buildHeap` on the unsorted vector instead of using `insert` N times.
- Runtime Analysis: O(N log N)
  - step 1 is O(N) if you use `buildHeap`
  - step 2: deleteMin is O(log N), and it's done N times, so it's O(N log N).