Recursion

Week 10

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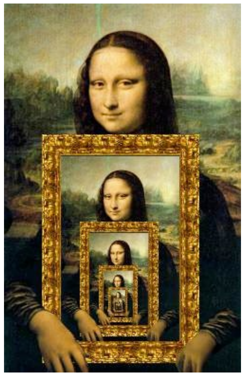
What is recursion?

- Generally, when something contains a reference to itself
- Math: defining a function in terms of itself
- Computer science: when a function calls itself:



```
void message() {
   cout << "This is a recursive function.\n";
   message();
}
int main() {
    message();
}
```

What happens when this is executed?

How can a function call itself?

- Infinite Recursion:

```
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
...
```


Recursive message() modified

- How about this one?

```
void message(int n) {
   if (n > 0) {
      cout << "This is a recursive function.\n";
      message(n-1);
   }
}
int main() {
    message(5);
}
```

Tracing the calls

- 6 nested calls to message:

```
message(5):
  outputs "This is a recursive function"
  calls message(4):
    outputs "This is a recursive function"
    calls message(3):
      outputs "This is a recursive function"
      calls message(2):
        outputs "This is a recursive function"
        calls message(1):
          outputs "This is a recursive function"
          calls message(0):
            does nothing, just returns
```

- depth of recursion (#times it calls itself) = 5

How to write recursive functions

- Branching is required (If or switch)
- Find a <u>base case</u>
  - one (or more) values for which the result of the function is **known** (no repetition required to solve it)
  - no recursive call is allowed here
- Develop the <u>recursive case</u>
  - For a given argument (say n), assume the function works for a smaller value (n-1).
  - Use the result of calling the function on n-1 to form a solution for n

Recursive function example

factorial

- Mathematical definition of n! (factorial of n)

```
if n=0 then    n! = 1
if n>0 then    n! = 1 x 2 x 3 x ... x n
```

- What is the base case?
  - n=0 (the result is 1)
- Recursive case: If we assume (n-1)! can be computed, how can we get n! from that?
  - n! = n * (n-1)!

Recursive function example:

- In C++:

```
int factorial(int n) {
   if (n==0)
      return 1;
   else
      return n * factorial(n-1);
}
```

- Tracing the calls to factorial:

```
factorial(4):
  return 4 * factorial(3);          =4 * 6 = 24
  calls factorial(3):
    return 3 * factorial(2);        =3 * 2 = 6
    calls factorial(2):
      return 2 * factorial(1);      =2 * 1 = 2
      calls factorial(1):
        return 1 * factorial(0);    =1 * 1 = 1
        calls factorial(0):
          return 1;
```

- Every call except the last makes a recursive call
- Each call makes the argument smaller

Recursive function patterns

- Recursive functions over integers look like this:

```
type f(int n) {
   if (n==0)
      //do the base case
   else
      // ...  f(n-1) ...
}
```

- Recursive functions over lists use the length/size of the list in place of n
  - base case: length=0 ==> empty list
  - recursive case: assume f works for list of length n-1, what is the answer for a list with one more element?

Recursive function example

sum of the list

- Recursive function to compute sum of a list of numbers
- What is the base case?
  - length=0 (empty list) => sum = 0
- If we assume we can sum the first n-1 items in the list, how can we get the sum of the whole list from that?
  - sum (list) = sum (list[0]..list[n-2]) + list[n-1]

Assume I am given the answer to this

Recursive function example

sum of a list (array)

```
int sum(int a[], int size) {  //size is number of elems
   if (size==0)
      return 0;
   else
      return sum(a,size-1) + a[size-1];
}
```

call sum on first n-1 elements   The last element

For a list with size = 4: sum(a,4)

```
sum(a,3) + a[3] =
sum(a,2) + a[2] + a[3] =
sum(a,1) + a[1] + a[2] + a[3] =
sum(a,0) + a[0] + a[1] + a[2] + a[3] =
0 + a[0] + a[1] + a[2] + a[3]
```


Recursive function example

count character occurrences in a string

- Recursive function to count the number of times a specific character appears in a string
- We will use the string member function `substr` to make a smaller string
  - `str.substr (int pos, int length);`
  - `pos` is the starting position in `str`
  - `length` is the number of characters in the result

```
string x = "hello there";
cout << x.substr(3,5);
```

lo th

- char access: `x[1]` is the second element ('e')

Recursive function example
count character occurrences in a string

- This example is different from how the book does it.

```
int numChars(char target, string str) {
   if (str.empty()) {
      return 0;
   } else {
      int result = numChars(target, str.substr(1,str.size()-1));
      if (str[0]==target)
         return 1+result;
      else
         return result;
   }
}

int main() {
  string a = "hello";
  cout << a << " " << numChars('l',a) << endl;
}
```


Recursive function example
greatest common divisor

- Greatest common divisor of two non-zero ints is the largest positive integer that divides the numbers evenly (without a remainder)
- This is a variant of Euclid's algorithm:

`gcd(x,y) = y` if y divides x evenly, otherwise:
`gcd(x,y) = gcd(y,remainder of x/y)`

- It's a recursive definition
- If x < y, then x%y is x (so gcd(x,y) = gcd(y,x))
- This moves the larger number to the first position.

Recursive function example
greatest common divisor

- Code:

```
int gcd(int x, int y) {
    cout << "gcd called with " << x << " and " << y << endl;
    if (x % y == 0) {
        return y;
    } else {
        return gcd(y, x % y);
    }
}

int main() {
    cout << "GCD(9,1): " << gcd(9,1) << endl;
    cout << "GCD(1,9): " << gcd(1,9) << endl;
    cout << "GCD(9,2): " << gcd(9,2) << endl;
    cout << "GCD(70,25): " << gcd(70,25) << endl;
    cout << "GCD(25,70): " << gcd(25,70) << endl;
}
```


Recursive function example
greatest common divisor

- Output:

```
gcd called with 9 and 1
GCD(9,1): 1
gcd called with 1 and 9
gcd called with 9 and 1
GCD(1,9): 1
gcd called with 9 and 2
gcd called with 2 and 1
GCD(9,2): 1
gcd called with 70 and 25
gcd called with 25 and 20
gcd called with 20 and 5
GCD(70,25): 5
gcd called with 25 and 70
gcd called with 70 and 25
gcd called with 25 and 20
gcd called with 20 and 5
GCD(25,70): 5
```

Recursive function example

Fibonacci numbers

- Series of Fibonacci numbers:

```
0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...
```

- Starts with 0, 1. Then each number is the sum of the two previous numbers

$$F_0 = 0$$
$$F_1 = 1$$
$$F_i = F_{i-1} + F_{i-2} \quad (\text{for } i > 1)$$

- It's a recursive definition

Recursive function example

Fibonacci numbers

- Code:

```
int fib(int x) {
    if (x<=1)        //takes care of 0 and 1
        return x;
    else
        return fib(x-1) + fib(x-2);
}

int main() {
    cout << "The first 13 fibonacci numbers: " << endl;
    for (int i=0; i<13; i++)
        cout << fib(i) << " ";
    cout << endl;

}
```

```
The first 13 fibonacci numbers:
0 1 1 2 3 5 8 13 21 34 55 89 144
```


Recursive function example

Fibonacci numbers

- Note: the recursive fibonacci functions works as written, but it is VERY inefficient.
- Counting the recursive calls to fib:

```
The first 40 fibonacci numbers:
fib (0)= 0  # of recursive calls to fib = 1
fib (1)= 1  # of recursive calls to fib = 1
fib (2)= 1  # of recursive calls to fib = 3
fib (3)= 2  # of recursive calls to fib = 5
fib (4)= 3  # of recursive calls to fib = 9
fib (5)= 5  # of recursive calls to fib = 15
fib (6)= 8  # of recursive calls to fib = 25
fib (7)= 13  # of recursive calls to fib = 41
fib (8)= 21  # of recursive calls to fib = 67
fib (9)= 34  # of recursive calls to fib = 109
...
fib (40)= 102,334,155  # of recursive calls to fib = 331,160,281
```


Recursive functions over linked lists

- Member functions of a linked list class can be defined recursively.
  - These functions take a pointer to a node in the list as a parameter
  - They compute the function for the list starting at the node p points to.
- The pattern:
  - base case: empty list, when p is NULL
  - recursive case: assume f works for list starting at p->next, what is the answer for the list starting at p? (it has one more element).

Recursive function example

count the number of nodes in a list

```
class NumberList
{
   private:
      struct ListNode  {
         double value;
         struct ListNode *next;
      };
      ListNode *head;
      int countNodes(ListNode *);  //private version, recursive

   public:
      NumberList();
      NumberList(const NumberList & src);
      ~NumberList();
      void appendNode(double);
      void insertNode(double);
      void deleteNode(double);
      void displayList();
      int countNodes();          //public version, calls private
};
```


Recursive function example

count the number of nodes in a list

```
// the private version, has a pointer parameter
// How many nodes are in the list starting at the pointer?
int NumberList::countNodes(ListNode *p) {
   if (p == NULL)
      return 0;
   else
      return 1 + countNodes(p->next);
}

// the public version, no arguments (Nodes are private)
// calls the recursive function starting at head
int NumberList::countNodes() {
   return countNodes(head);
}
```

Note that this function is overloaded

Recursive function example

display the node values in reverse order

```
// the private version, needs a pointer parameter
void NumberList::reverseDisplay(ListNode *p) {
   if (p == NULL) {
      //do nothing
   } else {
      //display the "rest" of the list in reverse order
      reverseDisplay(p->next);
      cout << p->value << " ";
   }
}

// the public version, no arguments
void NumberList::reverseDisplay() {
   reverseDisplay(head);
   cout << endl;
}
```


Sample Problems

isMember: Write a recursive Boolean function named isMember . The function should accept two arguments: an array and a value. The function should return true if the value is found in the array, or false if the value is not found in the array.

Linked List Sum : Write a function that accepts a linked list of integers. The function should recursively calculate the sum of all the numbers in the linked list. Demonstrate the function in a driver program.