Trees & Heaps

Week 12

Gaddis: 20
Weiss: 21.1-3

CS 5301
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Tree:
non-recursive definition

- **Tree**: set of nodes and *directed* edges
  - **root**: one node is distinguished as the root
  - Every node (except root) has exactly exactly one edge coming into it.
  - Every node can have any number of edges going out of it (zero or more).
- **Parent**: source node of directed edge
- **Child**: terminal node of directed edge
- **Binary Tree**: a tree in which no node can have more than two children.

Tree Traversals: examples

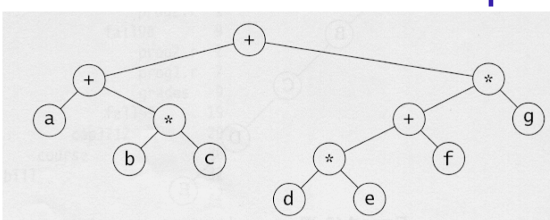


Figure 4.14 Expression tree for (a + b * c) + ((d * e + f) * g)

- **Preorder:** print node value, process left tree, then right `+ + a * b c * + * d e f g`
- **Postorder:** process left tree, then right, then print node value `a b c * + d e * f + g * +`
- **Inorder:** process left tree, print node value, then process right tree `a + b * c + d * e + f * g`

Binary Trees: implementation

- Structure with a data value, and a pointer to the left subtree and another to the right subtree.

```
struct TreeNode {
  <type> data;      // the data
  TreeNode *left;   // left subtree
  TreeNode *right;  // right subtree
};
```

- Like a linked list, but two “next” pointers.
- There is also a special pointer to the root node of the tree (like head for a list).

```
TreeNode *root;
```

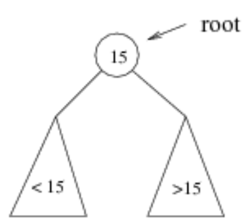
Binary **Search** Trees

- A special kind of binary tree, used for efficient searching, insertion, and deletion.
- Binary Search Tree property:
  For every node X in the tree:
  - All the values in the **left** subtree are **smaller** than the value at X.
  - All the values in the **right** subtree are **larger** than the value at X.
- Not all binary trees are binary search trees
- An inorder traversal of a BST shows the values in sorted order

Binary Search Trees: operations

- insert(x)
- remove(x) (or delete)
- isEmpty() (returns bool)
  - if the root is NULL
- find(x) (returns bool)
- findMin() (returns <type>)
  - Smallest element is found by always taking the left branch.
- findMax() (returns <type>)
  - Largest element is found by always taking the right branch.

BST: find(x)



Recursive Algorithm:

- if we are searching for 15 we are done.
- If we are searching for a key < 15, then we should search in the left subtree.
- If we are searching for a key > 15, then we should search in the right subtree.

BST: find(x)

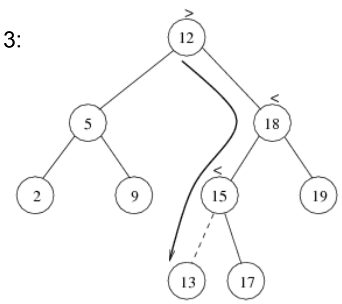
- Pseudocode
- Recursive

```
bool find (<type> x, TreeNode t) {

   if (isEmpty(t))
       return false                Base case

   if (x < value(t))
       return find (x, left(t))

   if (x > value(t))
       return find (x, right(t))

   return true  // x == value(t)

}
```

BST: insert(x)

- Algorithm is similar to find(x)
- If x is found, do nothing (no duplicates in tree)
- If x is not found, add a new node with x in place of the last empty subtree that was searched.

Inserting 13:



BST: insert(x)

- Pseudocode
- Recursive

```
void insert (<type> x, TreeNode t) {

   if (isEmpty(t))
       make t's parent point to new TreeNode(x)

   else if (x < value(t))
       insert (x, left(t))

   else if (x > value(t))
       insert (x, right(t))

   //else x == value(t), do nothing, no duplicates

}
```


Linked List example:

- Append x to the end of a singly linked list:
  - Pass the node pointer by reference
  - Recursive

```
void List::append (double x) {
   append(x, head);
}

void List::append (double x, Node *& p) {

   if (p == NULL) {
       p = new Node();
       p->data = x;
       p->next = NULL;
   }
   else
      append (x, p->next);
}
```

Public function

Private recursive function

BST: remove(x)

- Algorithm is starts with finding(x)
- If x is not found, do nothing
- If x is found, remove node carefully.
  - Must remain a binary search tree (smallers on left, biggers on right).

BST: remove(x)

- Case 1: Node is a leaf
  - Can be removed without violating BST property
- Case 2: Node has one child
  - Make parent pointer bypass the Node and point to that child

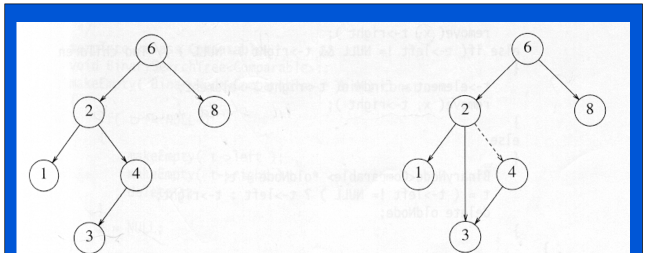


Figure 4.24 Deletion of a node (4) with one child, before and after

Does not matter if the child is the left or right child of deleted node

BST: remove(x)

- Case 3: Node has 2 children
  - Find minimum node in right subtree
    --cannot have left subtree, or it's not the minimum
  - Move original node's left subtree to be the left subtree of this node.
  - Make original node's parent pointer bypass the original node and point to right subtree

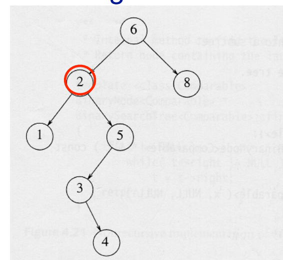
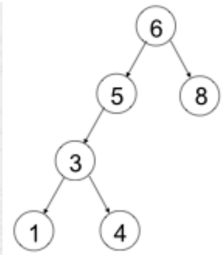


Figure 4.25 Deletion of a node (2) with two children, before and after



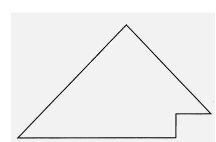
find minimum of 2's right subtree (3) and move 2's left subtree (1) to be it's new left. Remove 2 (make 6 point to 5).

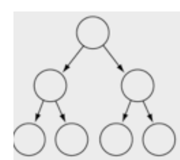
Binary heap data structure

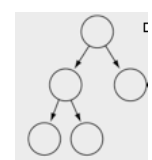
- A binary heap is a special kind of binary tree
  - has a restricted structure (must be complete)
  - has an ordering property (parent value is smaller than child values)
  - NOT a Binary Search Tree!
- Used in the following applications
  - Priority queue implementation: supports enqueue and deleteMin operations in O(log N)
  - Heap sort: another O(N log N) sorting algorithm.

Binary Heap:
structure property

- **Complete binary tree**: a tree that is completely filled
  - every level except the last is completely filled.
  - the bottom level is filled left to right (the leaves are as far left as possible).

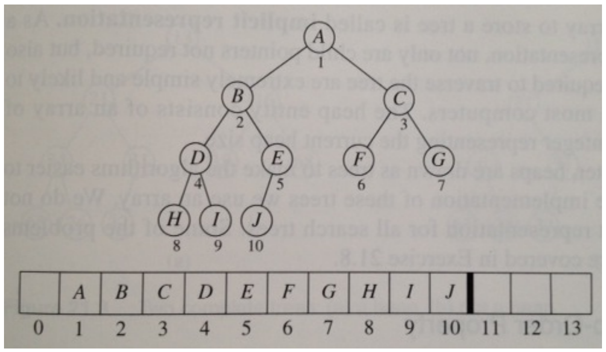






Complete Binary Trees

- A complete binary tree can be easily stored in an array
  - place the root in position 1 (for convenience)



Complete Binary Trees
Properties

- In the array representation:
  - put root at location 1
  - use an int variable (size) to store number of nodes
  - for a node at position i:
    - left child at position `2i` (if 2i <= size, else i is leaf)
    - right child at position `2i+1` (if 2i+1 <= size, else i is leaf)
    - parent is in position `floor(i/2)` (or use integer division)

Binary Heap:
ordering property

- In a heap, if X is a parent of Y, value(X) is less than or equal to value(Y).
  - the minimum value of the heap is always at the root.

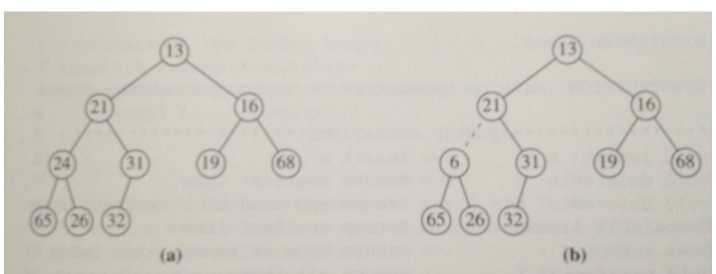


Figure 21.3 Two complete trees: (a) a heap; (b) not a heap.

Heap: insert(x)

- First: add a node to tree.
  - must be placed at next available location, size+1, in order to maintain a complete tree.
- Next: maintain the ordering property:
  - if x is greater than its parent: done
  - else swap with parent, repeat
- Called “percolate up” or “reheap up”
- preserves ordering property

Heap: insert(x)

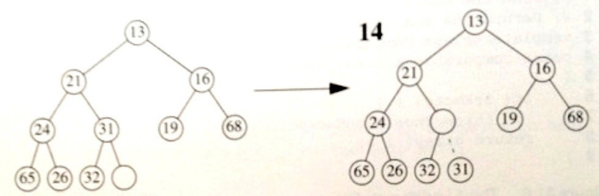


Figure 21.7 Attempt to insert 14, creating the hole and bubbling the hole up.

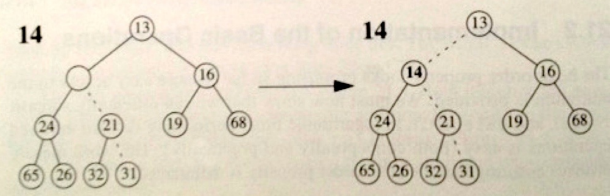


Figure 21.8 The remaining two steps required to insert 14 in the original heap shown in Figure 21.7.

Heap: deleteMin()

- Minimum is at the root, removing it leaves a hole.
  - The last element in the tree must be relocated.
- First: move last element up to the root
- Next: maintain the ordering property, start with root:
  - if no children, do nothing.
  - if one child, swap with parent if it's smaller than the parent.
  - if both children are greater than the parent: done
  - otherwise, swap the smaller of the two children with the parent, and repeat on that child.
- Called "percolate down" or "reheap down"
- preserves ordering property

Heap: deleteMin()

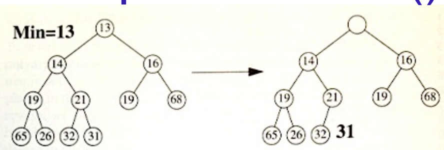


Figure 21.10 Creation of the hole at the root.

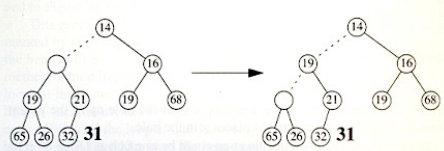


Figure 21.11 The next two steps in the deleteMin operation.

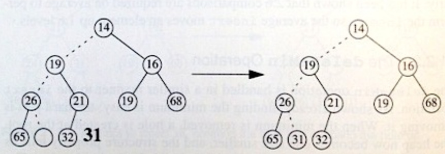


Figure 21.12 The Last two steps in the deleteMin operation.