Sets & Hash Tables

Week 13

Weiss: chapter 20

CS 5301
Fall 2016

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What are sets?

- A set is a collection of objects of the same type that has the following two properties:
  - there are no duplicates in the collection
  - the order of the objects in the collection is irrelevant.

- \{6,9,11,-5\} and \{11,9,6,-5\} are equivalent.

- There is no first element, and no successor of 9.

Set Operations

- Set construction
  - the empty set (0 elements in the set)
- isEmpty()
  - True, if the set is empty; false, otherwise.
- Insert(element)
  - If element is already in the set, do nothing; otherwise add it to the set
- Delete(element)
  - If element is not a member of the set, do nothing; otherwise remove it from the set.

Set Operations

- Member(element): boolean
  - True, if element is a member of the set; false, otherwise
- Union(Set1,Set2): Set
  - returns a Set containing all elements of the two Sets, no duplications.
- Intersection(Set1,Set2): Set
  - returns a Set containing all elements common to both sets.
Set Operations

- Difference(Set1, Set2): Set
  - returns a Set containing all elements of the first set except for the elements that are in common with the second set.
- Subset(Set1, Set2): boolean
  - True, if Set1 is a subset of Set2 (if all elements of the Set1 are also elements of Set2).

Implementation

- Array of elements implementation
  - each element of the set will occupy an element of the array.
  - the member (find) operation will be inefficient, must use linear search.

```java
class IntSet {
    int count;    // number of elements in the set, set to 0 in constr
    int intSet[100]; // stores the elements in positions 0..count
}

void insert(int x) {
    if (!member(x) && count<100) {
        intSet[count] = x;
        count++;
    }
}
```

- insert must not add duplicates:

```java
bool member(int x) {
    bool result = false;
    for (int i=0; i<count; i++) {
        if (intSet[i]==x) { return true; }
    }
    return false;
}
```

- Here is the set {Monday, Wednesday, Friday}:

<table>
<thead>
<tr>
<th>FALSE</th>
<th>TRUE</th>
<th>FALSE</th>
<th>TRUE</th>
<th>FALSE</th>
<th>TRUE</th>
<th>FALSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunday</td>
<td>0</td>
<td>Tuesday</td>
<td>2</td>
<td>Wednesday</td>
<td>3</td>
<td>Thursday</td>
</tr>
<tr>
<td>Friday</td>
<td>5</td>
<td>Saturday</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Exercise: implement all of the set operations for the IntSet.
Implementation

• Boolean array implementation
  - need a mapping function to convert an element of the universe to a position in the array

```java
int map(string x) {
  if (x=="Sunday") return 0;
  if (x=="Monday") return 1;
  if (x=="Tuesday") return 2;
  if (x=="Wednesday") return 3;
  if (x=="Thursday") return 4;
  if (x=="Friday") return 5;
  if (x=="Saturday") return 6;
}
```

- if `daysOfWeek[map("Monday")]]` is true, then Monday is in the Set.

What are hash tables?

• A Hash Table is used to implement a set (or a search table), providing basic operations in constant time (no loops/recursion):
  - insert
  - delete (optional)
  - find (also called “member”)
  - makeEmpty (need not be constant time)
• It uses a function that maps an object in the set (a key) to its location in the table.
• The function is called a hash function.

Using a hash function

HandyParts company makes no more than 100 different parts. But the parts all have four digit numbers.

This hash function can be used to store and retrieve parts in an array.

Hash(partNum) = partNum % 100

Use the hash function to place the element with part number 5502 in the array.
Placing elements in the array

Next place part number 6702 in the array.

\[
\text{Hash}(\text{partNum}) = \text{partNum} \mod 100
\]

\[
6702 \mod 100 = 2
\]

But values[2] is already occupied.

**COLLISION OCCURS**

<table>
<thead>
<tr>
<th>values</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>[0]</td>
<td>Empty</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[1]</td>
<td>4501</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[2]</td>
<td>5502</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[3]</td>
<td>7803</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[4]</td>
<td>Empty</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[97]</td>
<td>Empty</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[98]</td>
<td>2298</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[99]</td>
<td>3699</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

How to resolve the collision?

One way is by linear probing. This uses the following function

\[
(\text{HashValue + 1}) \mod 100
\]

repeatedly until an empty location is found for part number 6702.

Collision resolved

\[
(\text{Hash}(6702) + 1) \mod 100 = 3
\]

But values[3] is already occupied.

\[
(\text{Hash}(6702) + 2) \mod 100 = 4
\]

Part 6702 can be placed at the location with index 4.

<table>
<thead>
<tr>
<th>values</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>[0]</td>
<td>Empty</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[1]</td>
<td>4501</td>
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<td></td>
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</tr>
<tr>
<td>[2]</td>
<td>5502</td>
<td></td>
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<tr>
<td>[3]</td>
<td>7803</td>
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<td>[4]</td>
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</tr>
<tr>
<td>[97]</td>
<td>Empty</td>
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<tr>
<td>[98]</td>
<td>2298</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>[99]</td>
<td>3699</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Collision resolved

Part 6702 is placed at the location with index 4.

Where would the part with number 4598 be placed using linear probing?
Hashing concepts

- **Hash Table**: (usually an array) where objects are stored according to their key
  - **key**: attribute of an object used for searching/sorting
  - number of valid keys usually greater than number of slots in the table
  - number of keys in use usually much smaller than table size.

- **Hash function**: maps a key to a Table index

- **Collision**: when two separate keys hash to the same location

Implementation

- **Simple array implementation**
  - keys are ints, all greater than or equal to 0:

```cpp
class HashTable {
    private:
        int *array;            // array of int elements
        int size;              // size of array
        int hash (int key);  // maps key to position in array
    public:
        HashTable (int size);  //initialize all elements to -1
        ~HashTable();
        bool find(int);       //return true if int in table
        void insert (int);    //add int to table
        void display();       //show elements in table
    // do not implement remove
};
```

Collision Resolution: Linear Probing

- **Insert**: When there is a collision, search sequentially for the next open slot (-1)
  - Put the value in the table at that position
- **Find**: if the key is not at the hashed location, keep searching sequentially for it.
  - if it reaches an open slot (-1), the key is not found
- **Remove**: if the key is not at the hashed location, keep searching sequentially for it.
  - if the key is found, set the status to -1
- **Problem**: Removing an element in the middle of a chain. The Find method needs to know to keep searching to the end of the chain.
Collision Resolution: Separate Chaining

- Use an array of linked lists for the hash table
- Each linked list contains all objects that hashed to that location
  - no collisions

Hash function is still: \( h(K) = k \mod 10 \)

Implementation

- Array of linked lists implementation

  - The data structure:

    ```cpp
    class ChainedTable {
    private:
      static const int SIZE = 101;
      struct Node {
        int key;
        Node *nextNode;
      };
      Node* table[SIZE];  // array of pointers to Nodes
      int hash(int key) ;  // maps key to position in array
    public:
      ChainedTable();   // inits all pointers in array to NULL
      bool find(int);       // return true if int in table
      void insert (int);   // add int to table
      . . .
    }
    ```

Separate Chaining

- To insert an object:
  - compute hash(k)
  - if the object is not already in the list at that location, insert the object into the list.

- To find an object:
  - compute hash(k)
  - search the linked list there for the key of the object

- To delete an object:
  - compute hash(k)
  - search the linked list there for the key of the object
  - if found, remove it