Introduction to Recursion

Chapter 20.1-4

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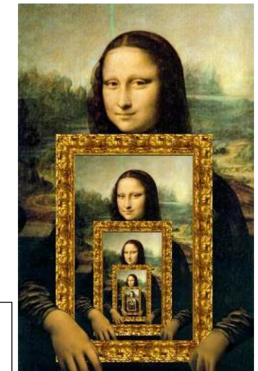
What is recursion?

- Generally, when something contains a reference to itself
- Math: defining a function in terms of itself
- Computer science: when a function calls itself:



```
void message() {
   cout << "This is a recursive function.\n";
   message();
}
int main() {
    message();
}
```

What happens when this is executed?

How can a function call itself?

- Infinite Recursion:

```
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
This is a recursive function.
...
```

Note: If you encounter infinite recursion in Lab, be sure to STOP your program BEFORE running it again!!!

Recursive message() modified

- How about this one?

```
void message(int n) {
    if (n > 0) {
       cout << "This is a recursive function.\n";
       message(n-1);
    }
}
int main() {
     message(5);
}
```

Tracing the calls

- 6 nested calls to message:

```
message(5):
  outputs “This is a recursive function”
  calls message(4):
    outputs “This is a recursive function”
    calls message(3):
      outputs “This is a recursive function”
      calls message(2):
        outputs “This is a recursive function”
        calls message(1):
          outputs “This is a recursive function”
          calls message(0):
            does nothing, just returns
```

- depth of recursion (#times it calls itself) = 5.

How to write recursive functions

- Branching is required (If or switch)
- Find a base case
  - one (or more) values for which the result of the function is **known** (no repetition required to solve it)
  - no recursive call is allowed here
- Develop the recursive case
  - For a given argument (say n), assume the function works for a smaller value (n-1).
  - Use the result of calling the function on n-1 to form a solution for n

Recursive function example
factorial

- Mathematical definition of n! (factorial of n)

```
if n=0 then    n! = 1
if n>0 then    n! = 1 x 2 x 3 x ... x n-1 x n
```

- What is the base case?
  - n=0 (the result is 1)
- Recursive case: If we assume (n-1)! can be computed, how can we get n! from that?
  - n! = n * (n-1)!

Recursive function example
factorial

```
int factorial(int n) {
   if (n==0)
      return 1;
   else
      return n * factorial(n-1);
}


int main() {
  int number;
  cout << “Enter a number “;
  cin >> number;
  cout << “The factorial of “ << number << “ is “
       << factorial(number) << endl;
}
```

Tracing the calls

- Calls to factorial:

```
factorial(4):
  return 4 * factorial(3);    =4 * 6 = 24
  calls factorial(3):
    return 3 * factorial(2);    =3 * 2 = 6
    calls factorial(2):
      return 2 * factorial(1);    =2 * 1 = 2
      calls factorial(1):
        return 1 * factorial(0);  =1 * 1 = 1
        calls factorial(0):
          return 1;
```

- Every call except the last makes a recursive call
- Each call makes the argument smaller

Recursive functions: ints and lists

- Recursive functions over integers follow this pattern:

```
type f(int n) {
   if (n==0)
      //do the base case
   else
      // ...  f(n-1) ...
}
```

- Recursive functions over lists (arrays, linked lists, strings) use the **length** of the list in place of n
  - base case: if (length==0) … // empty list
  - recursive case: assume f works for list of length n-1, compute the answer for a list with one more element.

Recursive function example

sum of the list

- Recursive function to compute sum of a list of numbers
- What is the base case?
  - length=0 (empty list) => sum = 0
- If we assume we can sum the first n-1 items in the list, how can we get the sum of the whole list from that?
  - sum (list) = sum (list[0]..list[n-2]) + list[n-1]

Assume I am given the answer to this

Recursive function example

sum of a list (array)

```
int sum(int a[], int size) {  //size is number of elems
   if (size==0)
      return 0;
   else
      return sum(a,size-1) + a[size-1];
}
```

call sum on first n-1 elements   The last element

For a list with size = 4:   sum(a,4) =

```
                      sum(a,3) + a[3] =
                 (sum(a,2) + a[2]) + a[3] =
           ((sum(a,1) + a[1]) + a[2]) + a[3] =
(((sum(a,0) + a[0]) + a[1]) + a[2]) + a[3] =
     0      + a[0]  + a[1]  + a[2]  + a[3]
```

Recursive function example
greatest common divisor

- Greatest common divisor of two non-zero ints is the largest positive integer that divides the numbers evenly (without a remainder)
- This is a variant of Euclid's algorithm:

  `gcd(x,y) = y` if `x/y` has no remainder otherwise:
  `gcd(x,y) = gcd(y,remainder of x/y)`

- It's a recursive definition, correctness is proven elsewhere.

Recursive function example
greatest common divisor

- Code:

```
int gcd(int x, int y) {
    if (x % y == 0) {
        return y;
    } else {
        return gcd(y, x % y);
    }
}

int main() {
    cout << "GCD(9,1): " << gcd(9,1) << endl;
    cout << "GCD(1,9): " << gcd(1,9) << endl;
    cout << "GCD(9,2): " << gcd(9,2) << endl;
    cout << "GCD(70,25): " << gcd(70,25) << endl;
    cout << "GCD(25,70): " << gcd(25,70) << endl;
}
```


Recursive function example
Fibonacci numbers

- Series of Fibonacci numbers:

  `0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...`

- Starts with 0, 1. Then each number is the sum of the two previous numbers

  $F_0 = 0$
  $F_1 = 1$
  $F_i = F_{i-1} + F_{i-2}$ (for $i > 1$)

- It's a recursive definition

```
int fib(int x) {
    if (x==0 || x==1)
        return x;
    else
        return fib(x-1) + fib(x-2);
}
```


Recursive function example
Fibonacci numbers

- Note: the recursive fibonacci functions works as written, but it is VERY inefficient.
- Counting the recursive calls to fib:

```
The first 40 fibonacci numbers:
fib (0)= 0  # of recursive calls to fib = 1
fib (1)= 1  # of recursive calls to fib = 1
fib (2)= 1  # of recursive calls to fib = 3
fib (3)= 2  # of recursive calls to fib = 5
fib (4)= 3  # of recursive calls to fib = 9
fib (5)= 5  # of recursive calls to fib = 15
fib (6)= 8  # of recursive calls to fib = 25
fib (7)= 13  # of recursive calls to fib = 41
fib (8)= 21  # of recursive calls to fib = 67
fib (9)= 34  # of recursive calls to fib = 109
...
fib (40)= 102,334,155  # of recursive calls to fib = 331,160,281
```