Searching, Sorting & Analysis

Unit 2

Chapter 8

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Definitions of Search and Sort

- Search: find a given item in an array, return the index of the item, or -1 if not found.
- Sort: rearrange the items in an array into some order (smallest to biggest, alphabetical order, etc.).
- There are various methods (algorithms) for carrying out these common tasks.
- Which ones are better? Why?

Linear Search

- Very simple method.
- Compare first element to target value, if not found then compare second element to target value . . .
- Repeat until:
  target value is found (return its index) or
  we run out of items (return -1).

Linear Search in C++

first attempt

```
int searchList (int list[], int size, int target) {

  int position = -1;            //position of target

  for (int i=0; i<size; i++)
  {
    if (list[i] == target)  //found the target!
      position = i;         //record which item
  }
  return position;
}
```

Is this algorithm correct (does it calculate the right value)?

Is this algorithm efficient (does it do unnecessary work)?

Linear Search in C++

second attempt

```
int searchList (int list[], int size, int target) {

  int position = -1;    //position of target
  bool found = false;   //flag, true when target is found

  for (int i=0; i < size && !found; i++)
  {
    if (list[i] == target)  //found the target!
    {
      found = true;         //set the flag
      position = i;         //record which item
    }
  }
  return position;
}
```

Is this algorithm correct (does it calculate the right value)?

Is this algorithm efficient (does it do unnecessary work)?

Program that uses linear search

```
#include <iostream>
using namespace std;

int searchList(int[], int, int);

int main() {
  const int SIZE=5;
  int idNums[SIZE] = {871, 750, 988, 100, 822};
  int results, id;

  cout << "Enter the employee ID to search for: ";
  cin >> id;

  results = searchList(idNums, SIZE, id);

  if (results == -1) {
    cout << "That id number is not registered\n";
  } else {
    cout << "That id number is found at location ";
    cout << results+1 << endl;
  }
}
```


Evaluating the Algorithm

- Does it do any unnecessary work?
- Is it time efficient?   How would we know?
- We measure time efficiency of algorithms in terms of number of main steps required to finish.
- For search algorithms, the main step is comparing an array element to the target value.
- Number of steps depends on:
  - size of input array
  - whether or not value is in array
  - where the value is in the array

Efficiency of Linear Search

how many main steps (comparisons to target)?

N is the number of elements in the array

	N=50,000	In terms of N
Best Case:	1	1
Average Case:	25,000	N/2
Worst Case:	50,000	N

Note: if we search for many items that are not in the array, the average case will be greater than N/2.

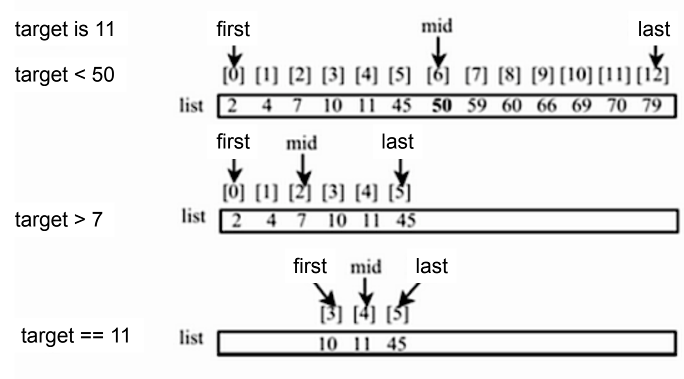
Binary Search

- Works only for SORTED arrays
- Divide and conquer style algorithm
- Compare target value to middle element in list.
  - if equal, then return its index
  - if less than middle element, repeat the search in the first half of list
  - if greater than middle element, repeat the search in last half of list
- If current search list is narrowed down to 0 elements, return -1

Binary Search Algorithm

example

We use first and last to indicate beginning and end of current search list



Binary Search in C++

```
int binarySearch (int array[], int size, int target) {

  int first = 0,        //index of beginning of search list
      last = size – 1,  //index of end of search list
      middle,           //index of midpoint of search list
      position = -1;    //position of target value
  bool found = false;     //flag

  while (first <= last && !found) {

    middle = (first + last) /2;    //calculate midpoint

    if (array[middle] == target) {
      found = true;
      position = middle;
    } else if (target < array[middle]) {
      last = middle – 1;         //search list = lower half
    } else {
      first = middle + 1;        //search list = upper half
    }
  }
  return position;
}
```

What if first + last is odd?
What if first==last?

Program using Binary Search

```
#include <iostream>
using namespace std;

int binarySearch(int[], int, int);

int main() {
  const int SIZE=5;
  int idNums[SIZE] = {100, 750, 822, 871, 988};
  int results, id;

  cout << "Enter the employee ID to search for: ";
  cin >> id;

  results = binarySearch(idNums, SIZE, id);

  if (results == -1) {
    cout << "That id number is not registered\n";
  } else {
    cout << "That id number is found at location ";
    cout << results+1 << endl;
  }
}
```

How is this program different from the one on slide 6?

Efficiency of Binary Search

Calculate worst case (target not in list) for N=1024

# Items left to search	# Comparisons so far
1024	0
512	1
256	2
128	3
64	4
32	5
16	6
8	7
4	8
2	9
1	10

Goal: calculate this value from N

$1024 = 2^{10}$ <==> $\log_2 1024 = 10$

Efficiency of Binary Search

If N is the number of elements in the array, how many comparisons (steps)?

$1024 = 2^{10}$ <==> $\log_2 1024 = 10$

$N = 2^{steps}$ <==> $\log_2 N = steps$

To what power do I raise 2 to get N?

	N=50,000	In terms of N
Best Case:	1	1
Worst Case:	16	$\log_2 N$

Rounded up to next whole number

Is Log_2N better than N?

Is binary search better than linear search?

Is this really a fair comparison?

Compare values of N/2, N, and Log_2 N as N increases:

N	N/2	Log_2N
5	2.5	2.3
50	25	5.6
500	250	9
5,000	2,500	12.3
50,000	25,000	15.6

N and N/2 are growing much faster than log N!

slower growing is more efficient (fewer steps).

8.3 Sorting Algorithms

- Sort: rearrange the items in an array into ascending or descending order.

- Bubble Sort
- Selection Sort

55 112 78 14 20 179 42 67 190 7 101 1 122 170 8 unsorted

1 7 8 14 20 42 55 67 78 101 112 122 170 179 190 sorted

The Bubble Sort

- On each pass:
  - Compare first two elements. If the first is bigger, they exchange places (swap).
  - Compare second and third elements. If second is bigger, exchange them.
  - Repeat until last two elements of the list are compared.
- Repeat this process (keep doing passes) until a pass completes with no exchanges

Bubble sort
Example: first pass

- 7 2 3 8 9 1 — 7 > 2, swap
- 2 7 3 8 9 1 — 7 > 3, swap
- 2 3 7 8 9 1 — !(7 > 8), no swap
- 2 3 7 8 9 1 — !(8 > 9), no swap
- 2 3 7 8 9 1 — 9 > 1, swap
- 2 3 7 8 1 9 — finished pass 1, did 3 swaps

Note: largest element is now in last position

Note: This is one complete pass!

Bubble sort
Example: second and third pass

- 2 3 7 8 1 9 — 2<3<7<8, no swap, !(8<1), swap
- 2 3 7 1 8 9 — (8<9) no swap
- finished pass 2, did one swap

2 largest elements in last 2 positions

- 2 3 7 1 8 9 — 2<3<7, no swap, !(7<1), swap
- 2 3 1 7 8 9 — 7<8<9, no swap
- finished pass 3, did one swap

3 largest elements in last 3 positions

Bubble sort
Example: passes 4, 5, and 6

- 2 3 1 7 8 9 — 2<3, !(3<1) swap, 3<7<8<9
- 2 1 3 7 8 9
- finished pass 4, did one swap
- 2 1 3 7 8 9 — !(2<1) swap, 2<3<7<8<9
- 1 2 3 7 8 9
- finished pass 5, did one swap
- 1 2 3 7 8 9 — 1<2<3<7<8<9, no swaps
- finished pass 6, no swaps, list is sorted!

Bubble sort
how does it work?

- At the end of the first pass, the largest element is moved to the end (it's bigger than all its neighbors)
- At the end of the second pass, the second largest element is moved to just before the last element.
- The back end (tail) of the list remains sorted.
- Each pass increases the size of the sorted portion.
- No exchanges implies each element is smaller than its next neighbor (so the list is sorted).

Bubble Sort in C++

```
void bubbleSort (int array[], int size) {

   bool swap;
   int temp;

   do {

      swap = false;
      for (int i = 0; i < (size-1); i++) {

         if (array [i] > array[i+1]) {

            temp = array[i];
            array[i] = array[i+1];
            array[i+1] = temp;
            swap = true;
         }
      }
   } while (swap);
}
```


Program using bubble sort

```
#include <iostream>
using namespace std;

void bubbleSort(int [], int);
void showArray(int [], int);

int main() {
   int values[6] = {7, 2, 3, 8, 9, 1};

   cout << "The unsorted values are: \n";
   showArray (values, 6);

   bubbleSort (values, 6);

   cout << "The sorted values are: \n";
   showArray(values, 6);
}

void showArray (int array[], int size) {
   for (int i=0; i<size; i++)
      cout << array[i] << " " ;
   cout << endl;
}
```

Output:

```
The unsorted values are:
7 2 3 8 9 1
The sorted values are:
1 2 3 7 8 9
```


Selection Sort

- There is a pass for each position (0..size-1)
- On each pass, the smallest (minimum) element in the rest of the list is exchanged (**swapped**) with element at the current position.
- The first part of the list (the part that is already processed) is always sorted
- Each pass increases the size of the sorted portion.

Selection sort

Example

- 7 2 3 8 9 1 — 1 is the min a[5], swap with a[0]
- 1 2 3 8 9 7 — 2 is the min a[1], self-swap a[1]
- 1 2 3 8 9 7 — 3 is the min a[2], self-swap a[2]
- 1 2 3 8 9 7 — 7 is the min a[5], swap with a[3]
- 1 2 3 7 9 8 — 8 is the min a[5], swap with a[4]
- 1 2 3 7 8 9 — sorted

Note: underlined portion of list is sorted.

Note: This is five passes

Selection Sort in C++

My version

```
// Returns the index of the smallest element, starting at start
int findIndexOfMin (int array[], int size, int start) {
   int minIndex = start;
   for (int i = start+1; i < size; i++) {
      if (array[i] < array[minIndex]) {
         minIndex = i;
      }
   }
   return minIndex;
}

// Sorts an array, using findIndexOfMin
void selectionSort (int array[], int size) {
   int temp;
   int minIndex;
   for (int index = 0; index < (size -1); index++) {
      minIndex = findIndexOfMin(array, size, index);
      //swap
      temp = array[minIndex];
      array[minIndex] = array[index];
      array[index] = temp;
   }
}
```

Note: saving the **index**

We need to find the **index** of the minimum value so that we can do the swap

Selection Sort in C++

compact version

```
void selectionSort(int array[], int size)
{
   for(int i=0; i<size; i++)
   {
      for(j=i+1; j<size; j++)
      {
         if(array[i]>array[j])
         {
            temp=array[i];
            array[i]=array[j];
            array[j]=temp;
         }
      }
   }
}
```

For each element, it scans the remainder of the array

If the next element is smaller than the element at the current position (i), then swap them

This finds the smallest element in the remainder of the list, and it ends up in position (i)

This version might do more swapping than the previous one

Program using Selection Sort

```
#include <iostream>
using namespace std;

int findIndexOfMin (int [], int, int);
void selectionSort(int [], int);
void showArray(int [], int);

int main() {
   int values[6] = {7, 2, 3, 8, 9, 1};

   cout << “The unsorted values are: \n”;
   showArray (values, 6);

   selectionSort (values, 6);

   cout << “The sorted values are: \n”;
   showArray(values, 6);
}

void showArray (int array[], int size) {
   for (int i=0; i<size; i++)
      cout << array[i] << “ ” ;
   cout << endl;
}
```

Output:

```
The unsorted values are:
7 2 3 8 9 1
The sorted values are:
1 2 3 7 8 9
```

Analysis of Algorithms
using Big O notation

- Which algorithm is better, linear search or binary search?
- Which algorithm is better, bubble sort or selection sort?
- How can we answer these questions?

- **Analysis of algorithms** is the determination of the amount of resources (such as time and storage) necessary to execute them.

Time Efficiency of Algorithms

- To classify the time efficiency of an algorithm:
  - Express "time" (using number of main steps), as a mathematical function of input size (or n below).

    Binary search: $f(n) = \log_2 (n)$
- Need a way to be able to compare these math functions to determine which is better.
  - We are mostly concerned with which function has smaller values (# of steps) at very large data sizes.
  - We compare the growth rates of the functions and prefer the one that grows more slowly.

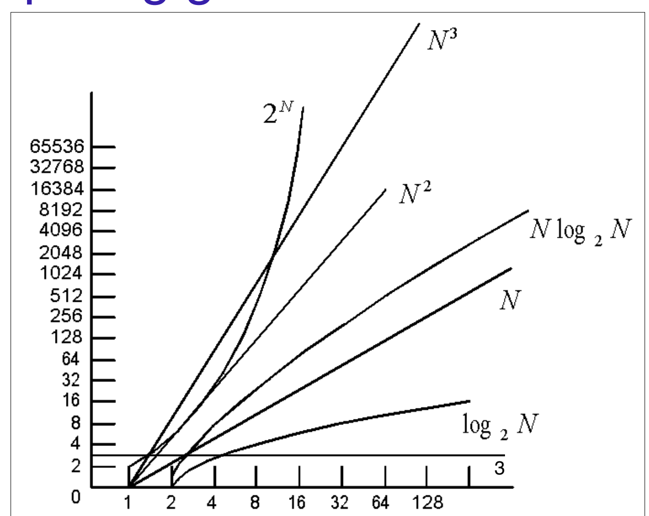
Classifications of (math) functions

Constant	$f(x)=b$	$O(1)$
Logarithmic	$f(x)=\log_b(x)$	$O(\log n)$
Linear	$f(x)=ax+b$	$O(n)$
Linearithmic	$f(x)=x \log_b(x)$	$O(n \log n)$
Quadratic	$f(x)=ax^2+bx+c$	$O(n^2)$
Exponential	$f(x)=2^x$	$O(2^n)$

- Last column is "big O notation", used in CS.
- It ignores all but dominant term, constant factors

Comparing growth of functions

Time
(# of steps)



Data size (N)

Time Efficiency of Algorithms

- To classify the time efficiency of an algorithm:
  - Express "time" (using number of main steps), as a mathematical function of input size.
  - Determine which classification the function fits into.
- Nearer to the top of the classification chart (on slide 31) is slower growth, and more efficient (constant is better than logarithmic, etc.)

Efficiency of Searches

(Assuming the array is already sorted)

- Linear Search, worst case:

  Linear search: $f(n) = n$ **O(N)**
- Binary Search, worst case:

  Binary search: $f(n) = \log_2 (n)$ **O(log N)**
- Which is slower growing (and thus fewer steps at large input sizes)?

  O(log N)
- Which search algorithm is more time efficient?

  Binary search

Efficiency of Selection Sort

- N is the number of elements in the list
- Outer loop executes N-1 times
- Inner loop executes N-1, then N-2, then N-3, ... then once. One comparison per loop iteration.
- Total number of comparisons (in inner loop):

$f(N) = (N-1) + (N-2) + \ldots + 2 + 1 =$ sum of 1 to N-1

sum of 1..N: $N + (N-1) + (N-2) + \ldots + 2 + 1 = N(N+1)/2$ $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

Subtract N from each side:

$$(N-1) + (N-2) + \ldots + 2 + 1 = N(N+1)/2 - N$$
$$= (N^2+N)/2 - 2N/2$$
$$= (N^2+N-2N)/2$$
$$= N^2/2 - N/2$$

O(N²)

Efficiency of Bubble Sort

- Each pass makes N-1 comparisons
- There will be (at most) N passes
- So worst case it's: $f(N) = (N-1)*N = N^2 - N$ **O(N²)**
- If you change the algorithm to look at only the **unsorted** part of the array in each pass, it's exactly like the selection sort:

  $(N-1) + (N-2) + \ldots + 2 + 1 = N^2/2 - N/2$ **still O(N²)**
- Neither algorithm is more efficient in the worst case.