A PROPOSED NEW BRAIN INJURY TOLERANCE CRITERION BASED ON THE EXCHANGE OF ENERGY BETWEEN THE SKULL AND THE BRAIN

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INTRODUCTION
The Head Injury Criterion limit, $HIC_{1000}=1000$, was developed based on skull fracture data (short time intervals $T$) and longer duration head translations that lead to Closed Head Injuries (CHI). However, recent results imply that the HIC limits depend on the time interval $T$ and the victim’s age [1]. Since the HIC lacks a clear physical interpretation, it is difficult to compare (verify) the new HIC limits with data from non-translational motions, e.g., to link these limits with the Diffuse Axonal Injury (DAI) criterion developed for head rotations [2]. The recent Head Impact Power (HIP) criterion can be applied to arbitrary motions, but it requires determining six parameters experimentally [3].

We believe that the CHI severity can be assessed more effectively by quantifying the way energy is exchanged between the skull and the brain (instead of using the total head impact power as the HIP does). Hence, we propose a novel Brain Injury Criterion (BIC), which is expressed in terms of the head’s motion and the brain’s surface geometry. Our criterion is rooted in the HIC formula but can be applied to arbitrary head motions, but it requires determining six parameters experimentally [3].

BIOPHYSICAL JUSTIFICATION OF BIC
The solid body dynamics allows determining the distribution of the kinetic energy in the moving skull. We use the time evolution of the kinetic energy per unit mass in the solid body layer surrounding the brain (including the dura layer) to compare the effects of arbitrary head motions. We refer to this layer, which approximates the brain’s outer-layer geometry, as the inner skull layer.

Arbitrary head motions (e.g., rotations) require a more subtle approach than translations. In a translated skull, the velocity is the same in the entire skull at a given time $t$ and the power per unit mass in the inner skull layer is proportional to the skull power. However, during an arbitrary skull motion, the power distribution near the brain surface may be highly uneven and the total (or average) energy exchange will not properly estimate the CHI probability since critically large energy amounts may be transferred locally.

Thus, to compare the consequences of arbitrary head motions, we propose to consider 2D cross-sections of the inner skull layer and use the maximum (taken over all cross-sections and intervals $T$) of a function of $T$ and energy/power that we derived from the HIC formula. With our approach, the rotation of a spherical skull about a horizontal axis (through the center) with the maximum tangential acceleration’s magnitude along the big circle equal to $a(t)$ should be comparable to translating the same skull by $a(t)$. Moreover, our approach intrinsically accounts for the energy that is exchanged between the skull and the brain via the falx cerebri (e.g., during rotations about a vertical axis).

In addition, we show that the HIC limits lead to a physical inconsistency if it is assumed that the power at the brain surface is proportional to the power within the inner skull layer. Consequently, the way energy is exchanged between the skull and the brain must be further studied, in particular to determine proper boundary conditions for the complex equations that describe the brain dynamics.
MATHEMATICAL FORMULATION OF BIC

Let \( v(x,t) \) be the magnitude of the skull’s velocity field at \( x=(x,y,z) \) and time \( t \). The integral of \( v(x,t)^2/2 \) over a thin strip of the inner skull layer along a cross-section divided by the strip’s area yields the average kinetic energy \( E(t) \) per unit mass in the strip, and \( P^2=E(t^2-E(t))/T \) is the absolute value of the average power per unit mass during time \( T=t_2-t_1 \). For translations, \( v \) depends only on \( t \) and \( E(t)=v^2/2 \).

If \( t \) is monotone, the average (over \( T \)) of the acceleration’s absolute value \( A=2^{1/2}[E(t)^{1/2}-E(t)^{1/2}]/T=2^{1/2}[E(t)^{1/2}-E(t)^{1/2}]/T=2^{1/2}P/E(t)^{1/2}E(t)^{1/2} \) and the HIC formula, \( HIC_{1000}=max|A|T \), can be expressed in energy terms. This allows us to extend the formula’s applicability to arbitrary head accelerations or decelerations and to introduce a new universal Brain Injury Criterion:

\[
BIC_{1000} = \max \left( \frac{\sqrt{2E(t)} - \sqrt{2E(t)^2}}{T^{\alpha-1}} \right) = \max \left( \frac{\sqrt{2P}}{\sqrt{E(t)} + \sqrt{E(t)}} \right)^{\alpha} T,
\]

where the maximum is taken over time intervals \( T \) and over all 2D cross-sections. \( \alpha \) is the parameter that allows us to replace the HIC limits, which vary with age, by fixed BIC limits. Indeed, assuming that \( \alpha \) depends on the age \( y \) as follows: \( \alpha(y \geq 6)=2.5, \alpha(3)=2.54, \alpha(1)=2.62 \), we find that the \( BIC_{15}=700 \) and \( BIC_{36}=1000 \) limits are in agreement (within several percent) with all six limits proposed in [1]. If the velocity \( v(t)=0 \) or \( v(t)^2=0 \), the BIC (and the HIC) formula reduces to:

\[
BIC_{1000}=HIC_{1000}=max|A|T^{\alpha-2/\alpha}.
\]

If the BIC (or the HIC) limit is assumed to be independent of \( T \), this formula implies that the limit power \( P_0=|A|^2T^{2/\alpha-2} \). By assuming that the average power \( P_0 \) at the brain surface inducing CHI is proportional to \( P_0 \), we get the unreasonable implication that for \( \alpha=2 \), less average power \( P_0 \) is needed for shorter intervals \( T \) at the brain surface to cause CHI. An acceptable formula is \( P_0=|A|^2T^{\beta} \), with \( \beta=0 \), which implies that \( P_0/P_0^T \cdot e^{-1/2} \), where \( e^{-1/2} \alpha \approx \beta \approx 0 \) quantifies how the power is modified during an exchange of energy between the brain and the skull in various intervals \( T \). If neither the BIC nor \( P_0 \) depends on \( T \), i.e., \( \beta=0 \), then obviously BIC-P0 (and HIC-P0), and the BIC has a simple physical interpretation—for any two motions with the same maximum \( P_0 \) obtained by considering all 2D cross-sections and intervals \( T \), the CHI severity should be similar, cf. how the power is correlated with the probability/severity of brain injuries in [3, 7]. The lower \( HIC_{15} \) limits in [1] suggest that the way energy is exchanged between the skull and the brain is governed by a more complex formula than \( T \).

NUMERICAL RESULTS

We numerically investigated how the brain dynamics changes when a traumatic (according to the HIC scale) head translation is replaced by a rotation with the same maximal energy exchange along a ‘proper’ brain surface cross-section. Both the K-V and N-L solutions exhibit a quite smooth transition when the axis of rotation is moved from the head’s center of mass to infinity (translation), indicating that a similar maximal energy exchange should lead to similar CHI.

We present here a comparison of (i) a forward translational head deceleration with triangularly-shaped \( a(t) \) and \( v(t)=0 \) corresponding to \( HIC_{36}=BIC_{36}=1000 \) with forward rotational head decelerations about the head’s center of mass, the skull’s base, and the neck; and (ii) a sideways translational head deceleration with clockwise rotational decelerations about vertical axes positioned inside and outside of the skull. The first (resp. second) case requires considering a sagittal (resp. horizontal) cross-section. Fig. 1 depicts the transition of the K-V solutions (in form of velocity fields at \( t=0.02s \)) in the first case, whereas Fig. 2 shows the transition based on the N-L model in the second case.

The sites within the brain characterized by strain values that can cause CHI vary with the axis position, but the maximal strain values remain similar. A variety of solutions is available in form of MPEG movies at http://www.csl.cornell.edu/~burtchs/CHI-research/.

ACKNOWLEDGMENT

We would like to thank Intel Corporation for donating the high-end Itanium 2 server that we used to perform some of our simulations.

REFERENCES