Erratum for "Mixed-Criticality Scheduling with Varying Processor Supply in Compositional Real-Time Systems"

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I. The Flaw

We are grateful to an anonymous reviewer who pointed out the following flaw in the schedulability analysis in [1].

In Lemma 2 in [1], we claimed that all virtual deadlines are met if

$$\sum_{\tau_i \in \mathcal{T}_{\mathrm{LO}}} u_i + \sum_{\tau_i \in \mathcal{T}_{\mathrm{HI}}} \frac{u_i}{x} \leq \beta^N, \tag{1}$$

where

$$\beta^N = \frac{\Theta^N}{\Pi}\left(1 - \frac{2(\Pi - \Theta^N)}{T^{\min}}\right). \tag{2}$$

This is incorrect. Because here we view each HI-task τ_i as a sporadic task τ_i' with a short period (and shorter relative deadline) $T_i' = x \cdot T_i$, the right-hand-side of (1) being β^N is not sufficient to ensure all virtual deadline are met — the $T^{\min}$ in (2) for the expression of β^N needs to be adjusted to $x \cdot T^{\min}$ for guaranteeing meeting all virtual deadlines. In Lemma 3 in [1], a similar flaw occurs with respect to β^C.

In repairing these flaws, we realized that the definitions of β^N and β^C are not good auxiliary parameters for notational simplicity anymore. Therefore, we introduced γ^N and γ^C as defined in the next section for the corrected schedulability test. The corrected analysis as well as other context and further results are later published in [2].

II. Corrected Schedulability Test

In this section, we present the corrected schedulability test for [1]. We match the indices of the lemmas and the theorem to that of their flawed counterpart in [1]. Also, Theorem 1 that was not affected by the flaws in [1] is used in the derivation. We denote

$$\gamma^N = \frac{2(\Pi - \Theta^N)}{T^{\min}}, \tag{3}$$

$$\gamma^C = \frac{2(\Pi - \Theta^C)}{T^{\min}_{\mathrm{HI}}}, \tag{4}$$

where

$$T^{\min} = \min_{\tau_i \in \mathcal{T}}\{T_i\}, \text{ and } T^{\min}_{\mathrm{HI}} = \min_{\tau_i \in \mathcal{T}_{\mathrm{HI}}}\{T_i\},$$

and set x by

$$x = \frac{U_{\mathrm{HI}} + w^N \gamma^N}{w^N - U_{\mathrm{LO}}}. \tag{5}$$

The following lemma shows that with x being set by (5), the deadlines of all (HI- and LO-) tasks are met in the nominal mode, since the virtual deadline of any job is no later than its actual deadline.

Lemma 2. *Given that $0 < x \leq 1$, all virtual deadlines of HI- and LO-tasks are met in the nominal mode if*

$$x \geq \frac{U_{\mathrm{HI}} + w^N \gamma^N}{w^N - U_{\mathrm{LO}}}.$$

Proof. In the nominal mode, treating the virtual deadlines as the actual deadlines, every HI-task τ_i can be viewed as a sporadic task τ_i' with a shorter period (and shorter relative deadline) $T_i' = x \cdot T_i$. Therefore,

$$u_i' = \frac{C_i}{T_i'} = \frac{C_i}{x \cdot T_i} = \frac{u_i}{x}.$$

On the other hand, the period, deadline, and therefore utilization of every LO-task remain unchanged. That is, each task $\tau_i \in \mathcal{T}$ can be viewed as a sporadic task with period T_i' such that

$$T_i' = \begin{cases} T_i, & \text{if } \tau_i \in \mathcal{T}_{\mathrm{LO}} \\ x \cdot T_i, & \text{if } \tau_i \in \mathcal{T}_{\mathrm{HI}}, \end{cases}$$

and therefore, given that $0 < x \leq 1$,

$$\begin{aligned} & \forall i, T_i' \geq x \cdot T_i \\ \Longrightarrow\ & \min_{\tau_i \in \mathcal{T}}\{T_i'\} \geq x \cdot T^{\min} \\ \Longrightarrow\ & \frac{2(\Pi - \Theta^N)}{\min_{\tau_i \in \mathcal{T}}\{T_i'\}} \leq \frac{2(\Pi - \Theta^N)}{x \cdot T^{\min}} \\ \Longrightarrow\ & 1 - \frac{2(\Pi - \Theta^N)}{\min_{\tau_i \in \mathcal{T}}\{T_i'\}} \geq 1 - \frac{2(\Pi - \Theta^N)}{x \cdot T^{\min}}. \end{aligned} \tag{6}$$

Furthermore, the budget supply in the nominal mode follows the periodic resource model with parameters (Π, Θ^N). Thus, by Theorem 1 in [1], the virtual deadlines of all tasks are met if

$$\begin{aligned}
&\sum_{\tau_i\in\mathcal{T}_{\text{LO}}} u_i + \sum_{\tau_i\in\mathcal{T}_{\text{HI}}} \frac{u_i}{x} \le \frac{\Theta^N}{\Pi}\left(1-\frac{2(\Pi-\Theta^N)}{\min_{\tau_i\in\mathcal{T}}\{T_i'\}}\right)\\
\overset{\text{by (6)}}{\Longleftarrow}\ &\sum_{\tau_i\in\mathcal{T}_{\text{LO}}} u_i + \sum_{\tau_i\in\mathcal{T}_{\text{HI}}} \frac{u_i}{x} \le \frac{\Theta^N}{\Pi}\left(1-\frac{2(\Pi-\Theta^N)}{x\cdot T^{\min}}\right)\\
\Longleftrightarrow\ & U_{\text{LO}} + \frac{U_{\text{HI}}}{x} \le w^N\cdot\left(1-\frac{\gamma^N}{x}\right)\\
\Longleftrightarrow\ & x \ge \frac{U_{\text{HI}}+w^N\gamma^N}{w^N-U_{\text{LO}}}.
\end{aligned}$$

The lemma follows. $\square$

Furthermore, the following lemma provides a sufficient condition for all actual deadlines of HI-tasks being met in the critical mode.

Lemma 3. *Given that* $0 < x \le 1$, *all actual deadlines of* HI-*tasks in the critical mode will be met if*

$$x \le 1 - \frac{U_{\text{HI}}+w^C\gamma^C}{w^C}.$$

Proof. At the mode switch point from the nominal to critical mode, a job from any task $\tau_i \in \mathcal{T}_{\text{HI}}$ must either be completed or have its virtual deadline at or after the mode switch point, because by Lemma 2, all virtual deadlines of HI-jobs are met in the nominal mode. That is, if not completed yet, a job of a HI-task τ_i must have an actual deadline at least $(1-x)T_i$ time units after this mode-switch point. Afterwards, any job from any task $\tau_i \in \tau_{\text{HI}}$ has at least T_i, which is greater than $(1-x)T_i$ (as $0 < x < 1.0$), time units from their releases in the HI-mode to their corresponding deadlines.

That is, in the critical mode, every HI-task τ_i can be viewed as a sporadic task τ_i' with a shorter period (and shorter relative deadline) $T_i' = (1-x)\cdot T_i$. Therefore,

$$u_i' = \frac{C_i}{T_i'} = \frac{C_i}{(1-x)\cdot T_i} = \frac{u_i}{1-x}.$$

Furthermore, the budget supply in the critical mode follows the periodic resource model with parameters (Π, Θ^C). Please note that (Π, Θ^C) specification has been actually maintained since time 0, as Θ^C denotes the *minimum* budget in each resource period, Therefore, any time intervals, including the ones in the critical mode can be viewed as a certain time interval under periodic resource model (Π, Θ^C). Thus, by Theorem 1 in [1], the actual deadlines of all HI-tasks are met in the critical mode if

$$\begin{aligned}
&\sum_{\tau_i\in\mathcal{T}_{\text{HI}}} u_i' \le \frac{\Theta^C}{\Pi}\left(1-\frac{2(\Pi-\Theta^C)}{\min_{\tau_i\in\mathcal{T}_{\text{HI}}}\{T_i'\}}\right)\\
\Longleftrightarrow\ &\sum_{\tau_i\in\mathcal{T}_{\text{HI}}} u_i' \le \frac{\Theta^C}{\Pi}\left(1-\frac{2(\Pi-\Theta^C)}{(1-x)\cdot T^{\min}_{\text{HI}}}\right)\\
\Longleftrightarrow\ & \frac{U_{\text{HI}}}{1-x} \le w^C\cdot\left(1-\frac{\gamma^C}{1-x}\right)\\
\Longleftrightarrow\ & 1-x \ge \frac{U_{\text{HI}}+w^C\gamma^C}{w^C}\\
\Longleftrightarrow\ & x \le 1-\frac{U_{\text{HI}}+w^C\gamma^C}{w^C}
\end{aligned}$$

Note that the second last step is because $0 < x < 1.0$. The lemma follows. $\square$

By Lemmas 2 and 3, the following theorem holds and serves as a sufficient schedulability test.

Theorem 2. *A mixed-criticality task set* $\mathcal{T}$ *is schedulable by EDF-VDVP on a VP with resource period* Π, *nominal budget* Θ^N, *and critical budget* Θ^C, *if*

$$\frac{U_{\text{HI}}+w^N\gamma^N}{w^N-U_{\text{LO}}} + \frac{U_{\text{HI}}+w^C\gamma^C}{w^C} \le 1. \tag{7}$$

Proof. Because it is clear that $\frac{U_{\text{HI}}+w^C\gamma^C}{w^C} > 0$, (7) implies $x < 1$ as x is defined by (5). Therefore, an additional explicit requirement of $x \le 1$ as discussed in Sec. **??** is redundant and can be omitted in this theorem. Furthermore, $0 < x \le 1$ implies the fact that the virtual deadline of every job is at or before its actual deadline. Thus, by Lemmas 2 and 3, the theorem follows. $\square$

References

[1] Kecheng Yang and Zheng Dong. Mixed-criticality scheduling with varying processor supply in compositional real-time systems. In *Proceedings of the 7th Workshop on Mixed Criticality Systems (WMC)*, 2019.

[2] Kecheng Yang and Zheng Dong. Mixed-criticality scheduling in compositional real-time systems with multiple budget estimates. In *Proceedings of the 41st IEEE Real-Time Systems Symposium (RTSS)*, 2020.